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All References should be given in the text by quoting the surname of the author, followed by year of publication, *e.g.*, (Roy, 1958). The full Reference should be given in a list at the end, arranged alphabatically, as follows; Roy, S. B., 1958, *Ind. J. Phys.*, **32,** 323.

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ON THE INFRARED SPECTRA OF MESITYLENE IN THE VAPOUR AND LIQUID STATES AND IN SOLUTIONS*

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ABSTRACT. The infrared absorption spectra of mesitylene in the vapour and liquid states and in solutions in different solvents have been investigated with a Perkin-Elmer Model 21 spectrophotometer and some of the assignments of the observed frequencies of the molecule made by previous workers have been critically examined. It has been observed that the frequencies due to a mode of aromatic CH vibration and the CH vibration in the methyl group are affected with the change from the vapour to liquid state. These frequencies are further affected in different ways when the compound is dissolved in different solvents. Attempts have been made to interpret these results.

INTRODUCTION

It was observed in some recent investigations that the 0–0 electronic transition is forbidden in the spectrum of mesitylene in the vapour state (Sponer and Stallcup, 1948), but this transition becomes allowed in the case of the liquid (Sen, 1959) and its solutions in some solvents (Roy, 1957). This was explained by the latter workers by assuming that the three-fold symmetry of the molecule is disturbed due to association of the molecules in the liquid state and in solutions. Information of such molecular association may be obtained from a comparative study of the infrared spectra of the substance in the vapour and liquid states and in solutions. Pitzer and Scott (1943) reported an analysis of the infrared spectrum of mesitylene, based on the data reported by previous workers. These data relate to the spectrum of the liquid or its solutions and it appears that no data are available for the vapour. Pitzer and Scott (1943) did not make any definite assignments of the vibrational frequencies of the molecule in the 3000 cm^{-1} region because of lack of suitable experimental data. It would be expected that in analysing these and other frequencies the study of infrared spectrum of mesitylene in the vapour state would be helpful. With these objects in view an investigation of the infrared spectra of mesitylene in the liquid and vapour states and in solutions in different solvents was undertaken and the results have been discussed in the present paper.

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* Read in the symposium on Raman and Infrared Spectra held at Nainital in October, 1959.

EXPERIMENTAL

Pure samples of mesitylene and of the solvents chloroform and carbon tetrachloride obtained from B.D.H. were used after being distilled under reduced pressure. The infrared spectra were recorded with a Perkin-Elmer Model 21 spectrophotometer with sodium chloride optics. The absorption cell used for studying the spectrum of the vapour consisted of a pyrex tube 10 cm long and connected through a side tube to a small bulb containing the liquid. Two plane parallel sodium chloride plates at the two ends of the tube, held in position with the help of suitable gaskets and screws, served as the windows. A similar evacuated tube was placed in the path of the reference beam. Absorption cells of different thicknesses varying from .025 mm. to 0.1 mm. were used for studying the spectra of the solutions and a much thinner film was used in the case of the pure liquid. Suitable compensation cells were put in the reference beam to eliminate the bands due to the solvents in the case of the solutions. The calibration of the spectrophotometer was checked by noting the position of the CO_2 band.

RESULTS AND DISCUSSION

The infrared absorption curves are reproduced in Figs. 1, 2, 3 and 4 and the frequencies of the observed bands are given in Table I.

TABLE I
Infrared spectrum of mesitylene
ν in cm^{-1}

Vapour	Liquid	5% Solution in $CHCl_3$	5% Solution in CCl_4
682 (s)	682	685	683
832 (vs)	835	836	835
925 (w)	925	915	920
1040 (ms)	1038	1038	1035
1238 (vw)	1200 (?)	1210	1230 (?)
1378 (ms)	1378	1378	1378
1440 (ms)	1442	1440	1450
1460 (s)	1470	1470	1472
1612 (vs)	1610	1605	1610
2882 (s)	2860	2858	2870
2950 (s)	2918	2918	2930
3050 (s)	3025	3005	3035

(a) *Assignment of the observed frequencies* :

It can be seen from Figs. 1(a) and 1(b) that in the vapour state mesitylene exhibits several bands which correspond to fundamental modes of vibration in the benzene ring as well as vibrations in the methyl group and that the spectrum shows two bands of moderate intensities at 3050 and 2950 cm^{-1} and a weaker band at 2882 cm^{-1}. Of these, the band at 3050 cm^{-1} is expected to be due to a CH vibration in the ring and the other bands to vibrations in the methyl group according to Pitzer and Scott (1943) who did not assign the band at 3050 cm^{-1} to any particular mode and also proposed that both the symme-
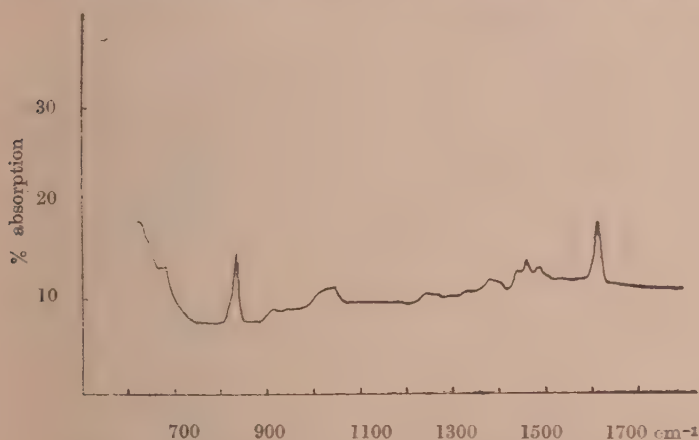
Fig. 1(a) Infrared spectrum of mesitylene (vapour at 26°C).
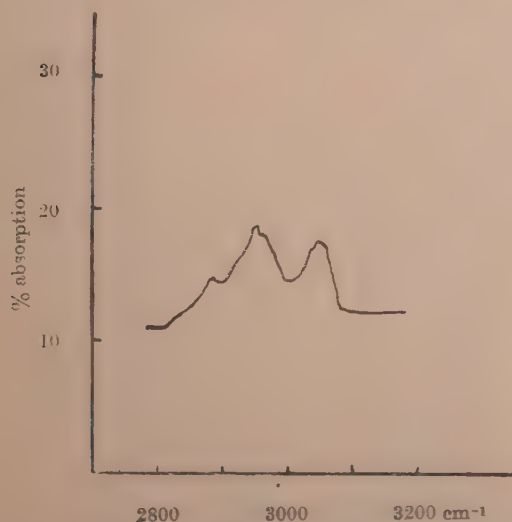
Fig 1(b) Infrared spectrum of mesitylene (vapour at 26°C).
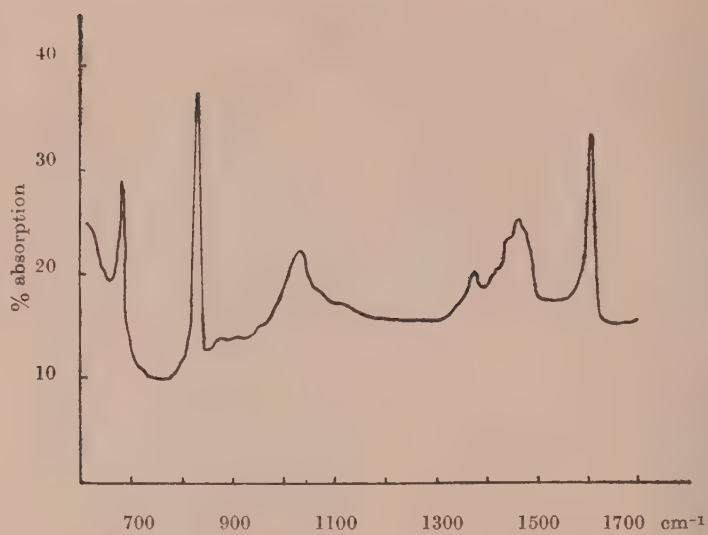
Fig. 2(a) Infrared spectrum of mesitylene (liquid at 26°C)
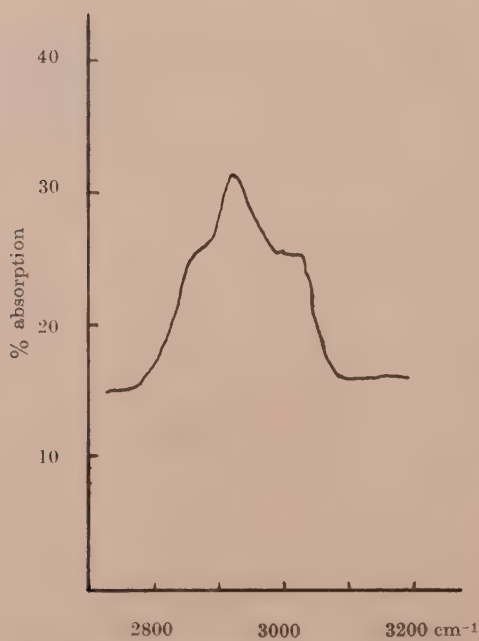
Fig. 2(b) Infrared spectrum of mesitylene (liquid at 26°C)
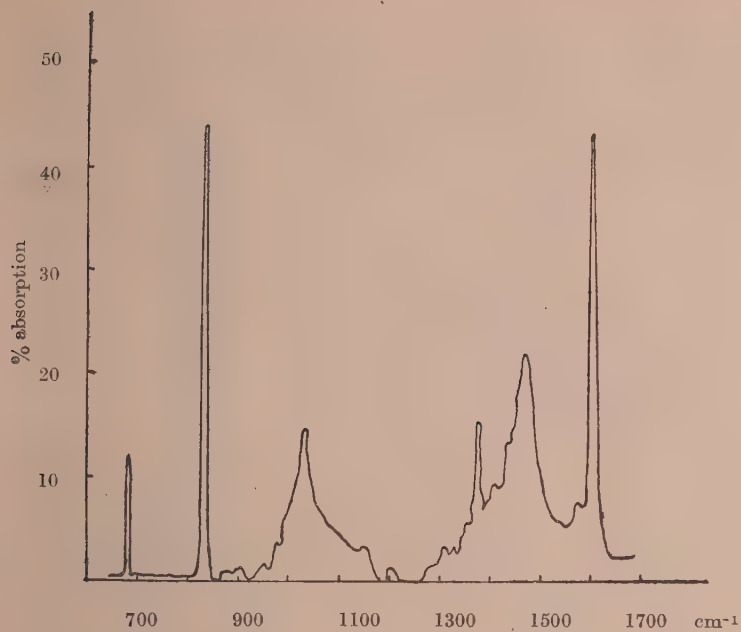
Fig. 3(a) Infrared spectrum of 5% solution of mesitylene in chloroform.
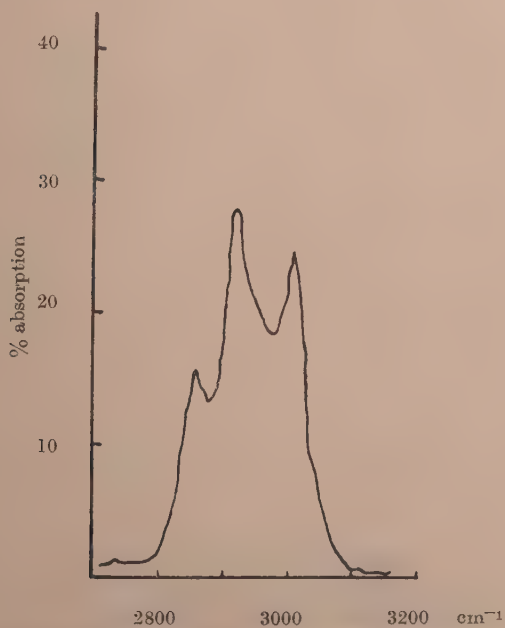
Fig. 3(b) Infrared spectrum of 5% solution of mesitylene in chloroform.
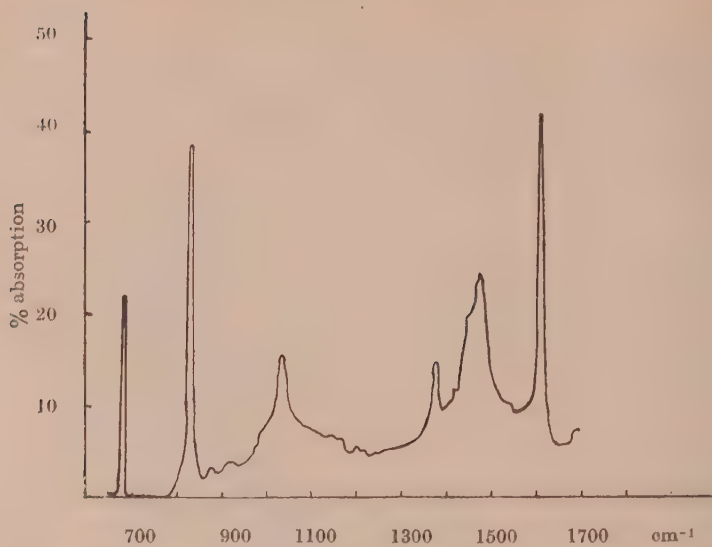
Fig. 4(a) Infrared spectrum of 5% solution of mesitylene in CCl₄
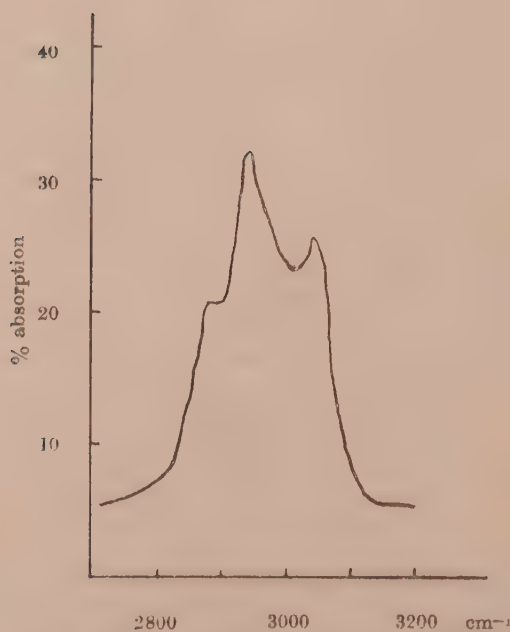
Fig. 4(b) Infrared spectrum of 5% solution of mesitylene in CCl₄.

tric and the asymmetric CH vibration in the methyl group would have the same frequency 2950 cm^{-1}. But from a review of the general trend of vibrational frequencies of the methyl group (Sheppard and Simpson, 1953) it appears that it would be proper to assign the band at 2950 cm^{-1} to asymmetric CH stretching vibration and that at 2882 cm^{-1} to the symmetric CH stretching vibration in the methyl group.
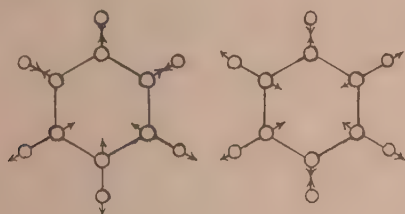
(a).   (b)
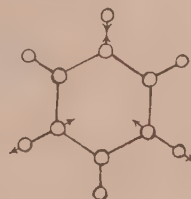
Fig. 5.   Fig. 6.

The large intensity of the band at 3050 cm^{-1}, which is expected to correspond to a mode of CH oscillation of the ring, points to the fact that it cannot be due to any mode of symmetric vibration. In assigning this band to an asymmetric mode of CH stretching vibration, the assignment of the bands in the infrared spectrum of 1,3,5-trideuterobenzene by Bailey *et al* (1946) has been followed. They have shown that the doubly degenerate e_{2g} and e_{2u} modes in benzene of frequencies 3047 and 3080 cm^{-1} shown in Figs. 5(a) and 5(b) respectively, interact with double decomposition into a nearly pure doubly degenerate stretching vibration of deuterium atom only and another almost pure doubly degenerate vibration of hydrogen atom only. In the present case of 1,3,5-trimethylbenzene similar argument would lead us to expect an almost pure CH vibration and another almost pure C–CH$_3$ vibration as shown in figure 6. In that case the band at 3050 cm^{-1} would represent a CH vibration of symmetry class e' (Baily *et al.*, 1946) and this band would be allowed in the infrared thus accounting for its observed intensity in the infrared spectrum. There is a weak band at 1238 cm^{-1} in the spectrum of the vapour, which might represent the corresponding C–CH$_3$ vibration also of e'-class.

On the long wavelength side there are several bands due to fundamental vibrations which include CH deformation vibrations within the methyl group. It is known from studies of infrared spectra of methylated compounds (Sheppard and Simpson, 1953) that in the methyl group there may be a symmetric deformation oscillation of frequency below 1400 cm^{-1} and two approximately doubly degenerate asymmetric modes of frequency near about 1450 cm^{-1}. Thus, it would be appropriate to attribute the band at 1378 cm^{-1} to the symmetric CH$_3$ bending, and the bands at 1440 and 1470 cm^{-1} to the two modes of asymmetric bending as done by Pitzer and Scott (1943). In addition to these there is a band

at 1040 cm^{-1} corresponding to wagging of the CH_3 group (Pitzer and Scott, **1943**; Sheppard *et al.*, 1953).

The other molecular frequencies may be assigned to different modes by following Bailey *et al* (1946) who analysed the spectrum of 1.3.5-trideuterobenzene. Pitzer and Scott (1943) also have analysed these bands. Thus the band at 682 cm^{-1} may be attributed to a C–C deformation perpendicular to the plane of the molecule (a_2''), that at 835 cm^{-1} to an out of plane hydrogen deformation (e') and the band at 1610 cm^{-1} to a mode of C–C stretching vibration (e').

(b) *Changes observed in the spectrum with the change from the vapour to liquid state and in solutions* :

As discussed above, in the spectrum of mesitylene vapour there is a band at 3050 cm^{-1} representing a CH stretching vibration in the benzene ring and two other bands at 2950 and 2882 cm^{-1} which represent respectively the asymmetric CH stretching vibrations in the methyl group. When the vapour is liquefied these bands are found to shift to 3025, 2918 and 2860 cm^{-1} respectively, while the positions of the other bands except that at 1238 cm^{-1} remain unchanged. In the case of the solution in chloroform the band at 3050 cm^{-1} is further shifted to 3005 cm^{-1}. When the substance is dissolved in the non-polar solvent carbon tetrachloride the three bands shift respectively to 2870, 2930 and 3035 cm^{-1} and the shifts are much smaller than those observed in the case of solution in chloroform. The presence of a permanent electric moment in the chloroform molecule is thus responsible for the large change in the frequencies.

The weak band at 1238 cm^{-1} which has already been assigned to a C–CH_3 vibrational mode in the vapour state is absent in the spectrum due to a thin film of the liquid. However, when a thicker film, of thickness of the order of ·025 mm., is used a very weak band at 1200 cm^{-1} is just perceptible. Also, in the case of solutions of chloroform and carbon tetrachloride a weak band at 1210 cm^{-1} and 1230 cm^{-1} respectively is observed, which may correspond to this mode of vibration. If the assignments of these bands due to the liquid and the solutions be correct then the C–CH_3 vibrational frequency also appears to be affected by change of state and environment. In this case also the change observed in the case of the solution in CCl_4 is small.

The shifts in the frequencies of the CH stretching vibration in the benzene ring and the CH stretching vibrations in the methyl group may indicate an association of the molecules in the liquid state through a virtual bond in which both the H-atoms of the ring and the CH_3 groups are involved. The changes in the intensity and frequency of the C–CH_3 vibration discussed above also support this conclusion. It is, however, unlikely that two CH_3 groups of neighbouring molecules form associated bonds to suppress the C–CH_3 vibration. Therefore, we have to postulate that such association is due to the influence of

π-electrons of the neighbouring molecules on the hydrogen atoms of the CH_3 group. This is also in conformity with the fact that the symmetry of the π-electron is disturbed and the 0–0 band appears in the ultraviolet absorption spectrum of the compound when the vapour is liquefied (Sen, 1959). That only the aromatic CH oscillation frequency is further affected in the case of the solution in chloroform shows that the chlorine atom is involved in an association only with the H-atom of the ring and not with the CH_3 group.

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A PRELIMINARY NOTE ON THE MAGNETIC ANISOTROPY AND SUSCEPTIBILITY OF Fe(NH₄SO₄)₂ . 6H₂O

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ABSTRACT. The theory of magnetic anisotropy and susceptibility of Fe^{2+} in Tutton salts has been worked out on the basis of Abragam and Pryces' method. It is found that the anisotropic part of the crystal field changes with temperature due to the thermal expansion of the crystal lattice. The spin-orbit coupling coefficient has to be decreased by 20% from its free ion value of -103 cm⁻¹ which indicates some amount of overlap between the $3d - Fe^{2+}$ and $s-$ and $p- O^{2-}$ charge clouds. The agreement of the theoretical values with the experiment is excellent.

INTRODUCTION

The five-fold degeneracy of the ground state $3d^6$, ^5D of Fe^{2+} ion in the free state is split up by the predominant cubic component of the crystal field into an orbital doublet and a triplet, the latter being lower by about 10^4 cm⁻¹, in the octahedrally co-ordinated Fe^{2+} salts, e.g., the Tutton salts (van Vleck, 1932). In these salts the octahedron of six water molecules surrounding each Fe^{2+} ion has very nearly a tetragonal symmetry with the z-axis elongated as observed from paramagnetic resonance measurements (Tinkham, 1955). This tetragonal component of the electric field and the spin-orbit coupling acting together partly removes the remaining degeneracy of the orbital levels inclusive of the five-fold spin degeneracy of each orbital level. The complete secular problem is highly complicated and a complete theory of the susceptibility of Fe^{2+} salts has not yet been worked out. In the case of the trigonal symmetry, the expression for the susceptibility has been approximately worked out by Pryce (1957) and in some details by Palumbo (1958), but the agreement with Jacksons (1959) measurements in $Fe SiF_6$, $6H_2O$ is not very satisfactry at all ranges of temperature.

We have derived the theoretical expressions for the magnetic susceptibility and anisotropy in the tetragonal case i.e., for the Fe^{2+} Tutton salts on the basis of Abragam and Pryces' theory (1951) and compared these with the experimental results on $Fe (NH_4SO_4)_2$, $6H_2O$ of Bose (1947) and Jackson (1924) and also with the resonance data of Tinkham.

Since the Fe^{2+} ion is in the D state, we have assumed $\alpha \approx \alpha' \approx 1$ (Abragam and Pryce) in this case. Under the action if the tetragonal field and the spin-orbit coupling, our fifteen order secular determinant breaks up into six $|+1>$, three $|0>$, four $|\pm 2>$ and two $|\pm 3>$ levels, one of the six $|\pm 1>$ levels, lying lowest. All these levels lie within a span of about 800 cm^{-1}. We next apply magnetic perturbation on these levels and get the expression for the magnetic susceptibility and finally for the square of the effective moment P^2_i $\frac{3k}{N\beta^2}$ (K_iT), i or $\perp$. The experimental value for the crystals are converted into ionic values of P^2_i taking two Fe^{2+} ions in the unit cell equally inclined to the crystallographic χ_1 axis, in the usual manner.

Here again we find that unless Δ is varied appreciably as in Ti^{3+} (1959a) and Co^{2+} (1959b) salts, the agreement with experiment at all temperatures is impossible. Thus the crystal field changes quite appreciably due to thermal expansion of the crystal lattice. The facts that at high temperatures the spin-lattice relaxation time is very small making paramagnetic resonance lines too wide to be observable and that it increases sufficiently at about 20°K to give well-resolved lines are also indications that the crystal field changes considerably with temperature. Moreover, the spin-orbit coupling coefficient has to be decreased by 20% from its free ion value of -103 cm^{-1}. This indicates a corresponding overlap between the $3d$ Fe^{2+} and s- and p- O^2 charge clouds.

TABLE I

Fe(NH$_4$SO$_4$)$_2$. 6H$_2$O ($\zeta = -80$ cm^{-1})

Temp. in °K	Δ cm^{-1}	$P^2_\|$	$P^2_\perp$	$P2 - P2$ $\perp$	g-values
300	650	36.26 (36.25)	26.31 (26.22)	9.95 (10.02)	—
160	. 500	44.12 (44.01)	24.72 (24.56)	19.39 (29.44)	—
90	400	51.80 (51.77)	21.28 (21.23)	30.51 (30.52)	—
20	270	—	—	—	$g_\| = 8.989$ (8.97±.02) $g_\perp = 0(0)$

The values in the parentheses indicate the mean of the experimental moment results of Jackson (1924) and Bose (1948). The g-values within parentheses are Tinkham's (1955).

The agreement with the magnetic anisotropies is quite excellent as can be seen in Table I, but the theoretical absolute P^2_i-values agree with the mean values of Jackson and Bose. The g-values given by Tinkham at 20°K also agree quite closely with our theoretical values.

The mathematical details will be published elsewhere shortly.

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INFRARED ABSORPTION SPECTRA OF DIAMONDS
OF DIFFERENT TYPES

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ABSTRACT. The infrared absorption spectra of ten specimens of diamond of which the fluorescence and absorption spectra had been studied earlier were investigated using a Perkin-Elmer Model 21 spectrophotometer. Two of these diamonds transparent upto about 2300 Å in the ultraviolet region and producing no fluorescence band at 4156 Å exhibit only very weak infrared absorption in different regions, while the other specimens show some or all the absorption bands reported by previous workers. It has been concluded from these results that these infrared bands excepting the band 1360 cm⁻¹ are due to impurities and are not produced by characteristic vibrations of the diamond lattice. The band 1360 cm⁻¹ has been assigned to the vibration in those portions of the lattice which are under strain due to presence of impurities.

INTRODUCTION

The infrared absorption spectra of a large number of diamonds were investigated by Robertson, Fox and Martin (1934) along with many other properties of the crystal and it was observed by them that all the specimens showed bands in the regions 2462–2491 cm⁻¹, 2167–2187 cm⁻¹, 2080–2106 cm⁻¹ and 1992–2015 cm⁻¹, but only the specimens of common type absorbing ultraviolet radiation beyond 3000 Å showed another set of bands in the region 1196–1387 cm⁻¹. These latter bands were absent in the spectra due to the crystals transparent to ultraviolet radiation beyond 3000Å. From these results they concluded that the diamonds could be divided into two types, e.g., Type 1 showing the absorption bands at 8μ and also absorption of ultraviolet rays beyond 3000Å and Type 2 not showing any absorption in these two regions. They, however, classified under Type 2 some diamonds which were transparent to ultraviolet radiation beyond 3000 Å but showed absorption bands in the region 2351Å–2335Å, as these diamonds also did not show any infrared absorption band in the region 8μ.

The study of the luminescence spectra of different specimens of diamond was later undertaken by Nayar (1941*a*, 1941*b*), Anna Mani (1944), Chandrasekharan (1948) and others. It was concluded by these workers that the luminescence is feeble in diamonds of Type 2 which show transparency in the ultraviolet region beyond 3000 Å and that the absorption band at 4156 Å and the

fluorescence band at this position were the characteristic bands of the diamond lattice. Bishui (1950) pointed out, however, that the fluorescence band at 4156 A exhibited by diamonds of Type 1 might be due to chemical impurities, as macroscopic strain in the crystal has no influence on the intensity of the band. He next studied (Bishui, 1952) quantitatively the intensity of the fluorescence band at 4156 Å relative to that of the Raman line at 1332 cm^{-1} and also the ultraviolet absorption spectra of eight selected specimens of diamond and observed that the intensity of the fluorescence band at 4156 A does not depend on the strength of the absorption band at the same place. He further observed that a specimen which is transparent to the visible region and to ultraviolet radiation upto 2270 A and shows only two absorption bands at 2360 Å and 2363.3 Å, produces strong fluorescence band at 4156 A, but those specimens which do not show any absorption bands in the visible or ultraviolet region upto 2240 A do not produce the fluorescence band at 4156 Å. From these results he concluded that diamonds of these totally transparent type are pure diamonds and should be classified under Type 2, while all other specimens showing fluorescence band at 4156 Å are of Type 1 and that the fluorescence band at 4156 Å is due to the impurity which produces the ultraviolet absorption bands at 2360 Å and 2363.3 Å.

The infrared spectra of these specimens of diamond were not known and it was thought worthwhile to study the infrared spectra to find out whether the band at 8μ was a characteristic band of diamonds of Type 1 which show the fluorescence band at 4156 Å and also whether the other bands observed in the infrared spectra of diamonds of Type 2 by Robertson, Fox and Martin (1934) are exhibited by all the diamonds of both the types.

EXPERIMENTAL

As mentioned above some of the specimens of diamond used previously by Bishui (1950, 1952) for investigating the fluorescence yield and ultraviolet absorption spectra were used in the present investigation. The specimens selected are D 1, D 4, D 5, D 6, D 7, D 8, D 9, D 10, D 11 and D 13. Of these, D 4 and D 13 were classified by him under Type 2 and the rest under Type 1.

A Perkin-Elmer Model 21 infrared spectrophotometer was used to study the absorption spectra. As the diamonds were smaller than the full apertures in the two beams in the spectrophotometer two small apertures of the same size made in two discs of black paper were used in the paths of the two beams, so that the radiation in the sample beam could enter into the spectrophotometer only after passing through the specimen of diamond covering the aperture in this beam. The size of the aperture was not less than 4.5 mm × 6 mm in any case and the smallest diamond used could wholly cover this aperture. Even when such a small aperture was used in the reference beam and the sample beam

was shut off the pen was driven to the position 1 instead of zero, so that the heights of the absorption peaks were not affected very much by the introduction of the apertures in the two beams.

RESULTS AND DISCUSSIONS

The positions of the bands observed in the infrared absorption spectra of the specimens of diamond are given in Table I. The intensities are given as strong, medium, etc. The details about the ultraviolet absorption limit, intensities of fluorescence and absorption band at 4156 Å and the dimensions of the specimens are given in Table II. The absorption curve due to D 6 is reproduced in Fig. 1. The curves due to D 9 and D 10 are shown in Fig. 2. The curves due to D 5, D 7 and D 10 show bands similar to those given by D 6 but with
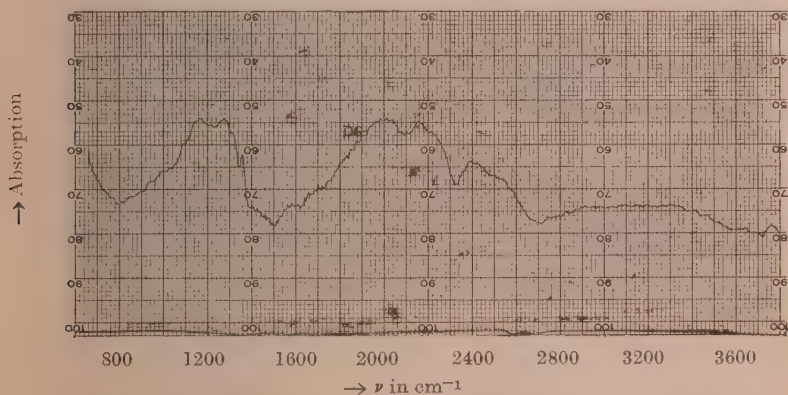
Fig. 1.   Infrared absorption spectrum of D 6
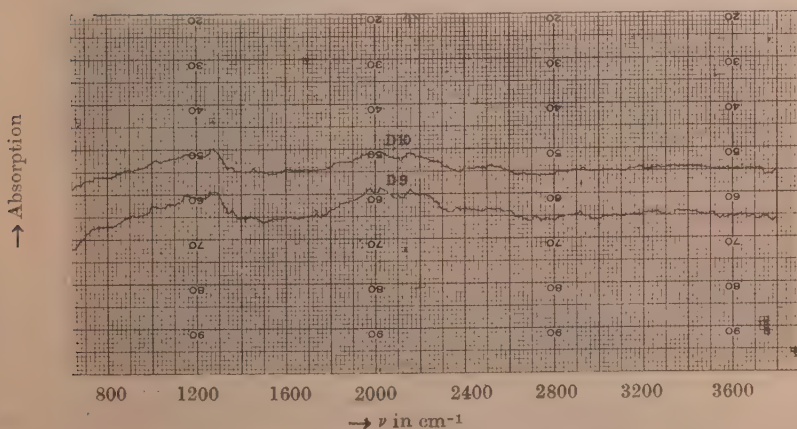
Fig. 2.   Infrared absorption spectra of D 9 and D 10

smaller strengths of absorption, and therefore, these curves have not been reproduced. Fig. 3 shows the curve due to D 1, which is a rose diamond belonging to Type 1, along with the curves given by D 4 and D 13 which are of Type 2. The curve due to D 8 was found to be similar to that due to D 1 and it has not been reproduced. Fig. 4 shows the curve due to D 11 which is transparent upto 2270Å but shows two absorption bands at 2360Å and 2363.3 Å respectively.
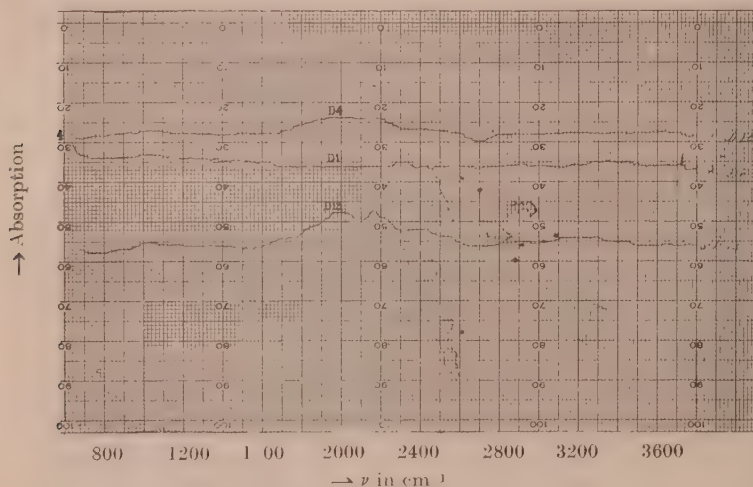
Fig. 3.   Infrared absorption spectra of D 1, D 4 and D 13
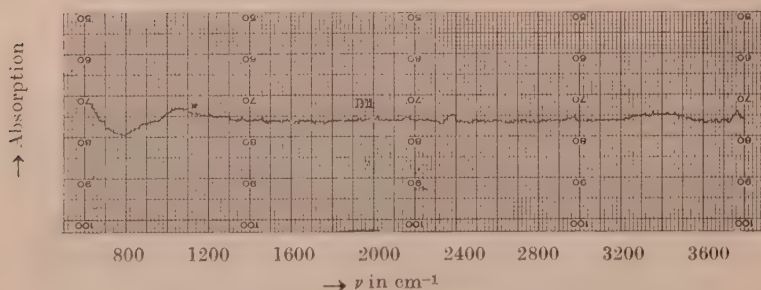
Fig. 4.   Infrared absorption spectrum of D 11

It can be seen from the figures and the tables mentioned above that the diamonds D 5, D 6, D 7, D 9 znd D 10 belonging to Type 1 produce similar absorption bands in the regions from 1000 cm⁻¹ to 1400 cm⁻¹ and from about 1900 cm⁻¹ to 2500 cm⁻¹. All these specimens show a sharp band at 1360 cm⁻¹ besides the other broad bands, but the height of this sharp peak is different for the different specimens. On the other hand, the diamonds D 1 and D 8 which

TABLE I

Infrared absorption bands of diamonds

Diamond No.	Frequencies of bands in cm^{-1} and intensities								
D 1	900 – 1300 (vw), (very broad band)								
D 4	—	—	—	1800 - 2200 (w), (very broad band)			—	—	—
D 5	1150 (m)	1270 (m)	1360 (vw)	2000 (mb)	2150 (mb)	2260 (w)	2400 (w)	2500 (w)	2900-3400 (wb)
D 6	1160 (s)	1280 (s)	1360 (m)	2000 (s,b)	2160 (s)	2240 (m)	2400 (w)	2500 (w)	2900-3400 (w)
D 7	1180 (m)	1280 (s)	1360 (w)	2000 (s,b)	2160 (s)	2220 (m)	—	2500 (w,b)	3000-3400 (vw)
D 8	900–1400 (w,vb)								
D 9	1000 (w)	1160 (m)	1280 (m)	1360 (vw)	1980 (m)	2020 (m)	2160 (m)	2220 (w)	—
D 10	1000 (w)	1160 (m,b)	1280 (m)	1360 (vw)	1960 (m)	2020 (m)	2160 (m)	—	—
D 11				1950 (w)	2000 (w)	2150 (w)	2400 (vw)		
D 13				2000 (w,b)	2150 (w)	2240 (vw)	2400 (vw)		

TABLE II

Properties of the specimens of diamond used

Diamond No.	Dimensions in mm.	Intensity of absorption band at 4152 Å at −180°C	Intensity of fluorescence band at 4152 Å at −180°C	Ultraviolet absorption limit	
D 1	$9 \times 7 \times 1.5$	strong	very strong	opaque beyond 3500 Å	
D 4	$11 \times 8 \times 1.5$ (triangular)	zero	very weak	limit	2280 Å
D 5	$10 \times 8 \times 2$	weak	very strong	,,	3000 Å
D 6	$14 \times 12 \times 2$	medium	very strong	,,	3500 Å
D 7	$9 \times 7.5 \times 1.35$	weak	strong	,,	2560 Å
D 8	$9 \times 5.5 \times 1.3$	medium	weak	,,	3000 Å
D 9	$6 \times 5 \times 0.8$	weak	very strong	,,	2550 Å
D 10	$7.5 \times 5.5 \times 1.09$	very weak	strong	,,	2810 Å
D 11	$7 \times 5 \times 0.95$ (low pyramid)	weak	very strong	,, bands at 2360 & 2363.3 Å	2270 Å
D 13	$8 \times 5 \times 0.84$ (low pyramid)	zero	zero	limit	2240 Å

also are of Type 1 and absorb all ultraviolet radiation of wavelengths shorter than 3000 Å, do not produce any discrete infrared absorption bands in the region from 2000 cm⁻¹ to 3500 cm⁻¹, but each of them shows only a very weak continuous absorption in the region 900–1400 cm⁻¹. The specimens D 4 and D 13 which belong to Type 2 and are transparent upto 2240 Å do not show any absorption band in the region 900–1400 cm⁻¹, but although D 13 produces a few weak and broad bands in the region 2000–2400 cm⁻¹, D 4 shows only very weak continuous absorption in the region 1800–2200 cm⁻¹. The specimen D 11 produces strong fluorescence band at 4156 Å and unlike other diamonds of Type 1 it is transparent up to about 2240 Å but shows two absorption bands at 2360 Å and 2363.3 Å. It exhibits no absorption bands in the region 1000 cm⁻¹—1400 cm⁻¹.

It can be seen from the above results that all the diamonds opaque to ultraviolet radiation beyond 3000 Å and classified under Type 1 do not necessarily produce infrared absorption bands in the regions 2000–2260 cm⁻¹ and 2400 cm⁻¹–2500 cm⁻¹. D 1 and D 8 are examples of such diamonds not showing any infrared bands in these regions. Again, D 11, although classified under Type 1, does not show any band in the region 1000–1400 cm⁻¹. Thus the intensity of the fluorescence band at 4156 Å cannot be correlated with infrared

absorption in any of the regions mentioned above. The fluorescence band was, however, ascribed to some impurities which produce the ultraviolet absorption bands at 2360 Å and 2363.3 A by Bishui (1952). It appears from these facts that the infrared absorption in the regions shown in Table I are due to some impurities and they do not represent the characteristic frequencies of vibration of the diamond lattice.

It has to be pointed out, however, that the band at 1360 cm^{-1} is much sharper than the other bands and its frequency is also very near to the frequency of the Raman line of diamond. The vibration of frequency 1332 cm^{-1} giving the Raman line is forbidden in the infrared spectra. It appears, however, that in small microscopic regions in the diamond in which impurities are present the symmetry of the lattice is destroyed and the mode of vibration is made allowed with slightly increased frequency. The intensity of the infrared band produced in this way will depend on the percentage of the total volume of the crystal which is under strain due to the presence of the impurities. This may explain the difference in the intensities of this band observed in the different cases. The strength of the absorption at the 1360 cm^{-1} band is in conformity with the fact that even a 0.025 mm thick film of 1% solution of any organic liquid produces large peaks in the infrared absorption spectra.

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ON THE VARIATION OF THE TRANSPORT FACTOR OF A JUNCTION TRANSISTOR WITH INJECTED CARRIER CONCENTRATION

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ABSTRACT. An attempt has been made to set up a general equation governing the distribution of injected carriers in the base region of a p–n–p junction transistor and hence to obtain an expression for the transport factor β. Subject to certain approximations, a relation is derived giving the emitter current density as a function of the concentration of injected carriers and with its help, the transport factor is expressed explicitly in terms of the latter. The expression is critically examined in the light of recombination process both on the surface and in the volume. The results are compared with those suggested by previous workers.

The possible effect of the presence of a significant electronic component of current, across the emitter-base junction on the expression for the transport factor is also considered. It is shown that the effect if any would be very small. An electronic component of current however, affects the value of the current amplification factor α and a categorical experimental verdict in favour of one or the other of the different possible modes of recombination is not possible unless the so-called emitter efficiency term can be determined by independent experimental measurement.

1. INTRODUCTION

The current amplification factor of a junction transistor, as given approximately by the product of the so-called transport factor β and the emitter efficiency γ, is known to vary with the emitter current I_e. A large amount of work (Webster 1954, Rittner 1954, Giacoletto 1955, Misawa 1955, Fletcher 1956, Hauri 1956, Matz, 1598 and Kaufmann 1959) has already been done to account for this variation but a satisfactory answer has not yet been obtained. One reason for this is the fact that a rigorous solution of the diffusion equation of a junction transistor holding for all values of I_e is difficult to obtain and only approximate solutions for operation at high injection level have to be inferred by making use of the known results for that at low level and of certain plausible assumptions. The deductions are obviously rather crude and it is not known whether these may be applicable to the intermediate levels of operation. An attempt was therefore made to set up a general equation governing the distribution of carriers injected in the base region with a view particularly to obtain an expression for the transport factor β valid over a wide range of operation. In the present paper, an account

is given of the results thus obtained for three cases of general interest. viz., (i) when recombination of carriers is confined only to the surface; (ii) when recombination occurs only in the volume and (iii) when recombination occurs both on the surface and in the volume. The results are compared with the values given by the previous workers.

2. GENERAL EQUATION FOR THE DISTRIBUTION OF CARRIERS IN THE BASE REGION

The basic equations governing the one dimensional flow of minority carriers through the base region of a junction transistor are the following :

$$J_n = ne\mu_n E + eD_n \frac{dn}{dx} \qquad \qquad \dots \ (1)$$

$$J_p = pe\mu_p E + eD_p \frac{dp}{dx} \qquad \qquad \dots \ (2)$$

where J_n, J_p = electron and hole current densities across the emitter-base boundary,

       n, p = electron and hole concentrations in the base region,

       μ_n, μ_p = electron and hole mobilities,

       D_n, D_p = diffusion constants for electrons and holes,

       E = electric field in the base region,

and e = electronic charge.

Confining to transistors of the p-n-p type we note that for high values of emitter efficiency, $J_n \approx 0$. The electric field E can now be eliminated (Webster 1954) between Eqs. (1) and (2) giving

$$J_p = -eD_p \left(1 + \frac{p}{p+N_d} \right) \frac{dp}{dx} \qquad \qquad \dots \ (3)$$

where N_d = equilibrium donor concentration in the base region.

When recombination is present, the time rate of decay of injected hole density in the base region is given by

$$\frac{dp}{dt} = -\frac{p-p_B}{\tau} - \frac{1}{e}\frac{dJ_p}{dx} \qquad \qquad \dots \ (4)$$

where p_B = thermal equilibrium value of hole density in the base region, and τ = effective lifetime of holes in the base region.

Eliminating J_p from Eqs.(3) and (4) one obtains

$$\frac{dp}{dt} = -\frac{p-p_B}{\tau} + D_p \left(1 + \frac{p}{p+N_d} \right) \frac{d^2p}{dx^2} + D_p \frac{N_d}{(p+N_d)^2} \cdot \left(\frac{dp}{dx} \right)^2 \quad \dots \ (5)$$

Eq. (4) assumes tacitly that the lifetime τ is independent of the injected minority carrier concentration. This assumption is not, however, generally true. Experimental results on the variation of τ with I_e are somewhat conflicting. Results of a recent work (Deb and Daw, 1958) show that in general, leaving out the case involving very low values of I_e, the effective lifetime τ passes through a maximum as the injection level is increased from a low value. An analytical relation suggested to account for this variation is

$$\frac{1}{\tau} = \frac{1}{\tau_B}\left(1+\frac{p}{N_d}\right) + \frac{\nu_B\left(1+\dfrac{p}{N_d}\right)}{1+\dfrac{2p}{N_d}} . \qquad \dots \quad (6)$$

where $\tau_B =$ volume recombination lifetime when $p \ll N_d$,

and $1/\nu_B =$ surface recombination lifetime when $p \ll N_d$,

it being assumed that the volume recombination is bimolecular. Accepting Eq.(6) one obtains from Eq.(5),

$$\frac{dp}{dt} = -(p-p_B)\left\{\frac{1}{\tau_B}\left(1+\frac{p}{N_d}\right) + \frac{\nu_B\left(1+\dfrac{p}{N_d}\right)}{1+\dfrac{2p}{N_d}}\right\} + D_p\left(1+\frac{p}{p+N_d}\right)\frac{d^2p}{dx^2}$$

$$D_p\left(\frac{dp}{dx}\right)^2\frac{N_d}{(p+N_d)^2} \qquad \dots \quad (7)$$

Eq. (7) is the general form of diffusion equation valid for all values of I_e, provided the lifetime τ follows the relation given by Eq.(6).

3. SOLUTION OF DIFFUSION EQUATION AND GENERAL EXPRESSION FOR THE TRANSPORT FACTOR

We now proceed to obtain a solution of Eq. (7) deduced in the preceding section. For simplicity, we assume the steady state operating condition. For this, $\dfrac{dp}{dt} = 0$ and

$$\left(1+\frac{p}{p+N_d}\right)\frac{d^2p}{dx^2} + \frac{N_d}{(p+N_d)^2}\left(\frac{dp}{dx}\right)^2 - \frac{1}{D_p}\left\{\frac{1}{\tau_B}\left(1+\frac{p}{N_d}\right) + \frac{\nu_B\left(1+\dfrac{p}{N_d}\right)}{1+\dfrac{2p}{N_d}}\right\}$$

$$= 0 \qquad \dots \quad (8)$$

This equation is of a standard form and can be integrated easily giving

$$Z \left(\frac{2p+N_d}{p+N_d} \right)^2 = \frac{2}{D_p} \int \left\{ \frac{2p+N_d}{\tau_B N_d} (p-p_B) + \nu_B (p-p_B) \right\} dp + C$$

$$= \frac{2}{D_p \tau_B} \left[\frac{2p^3}{3N_d} + \left(\frac{1}{2} + \frac{p_B}{N_d} \right) p^2 - p \cdot p_B \right] + \frac{2\nu_B}{D_p} \left[\frac{p^2}{2} - p \cdot p_B \right] + C \quad \dots \quad (9)$$

where $Z = \left(\dfrac{dp}{dx} \right)^2$ and C is the constant of integration.

Let us now introduce the variable $y = \dfrac{p}{N_d}$ so that

$$Z = \left(\frac{dp}{dx} \right)^2 = N_d^2 \left(\frac{dy}{dx} \right)^2 \qquad \dots \quad (10)$$

Substituting Eq. (10) in Eq. (9),

$$\left(\frac{dy}{dx} \right)^2 \left(\frac{1+2y}{1+y} \right)^2 = \frac{2}{D_p \tau_B} \left[\frac{2}{3} y^3 + \left(\frac{1}{2} + \frac{p_B}{N_d} \right) y^2 - \frac{p_B}{N_d} \cdot y \right]$$

$$+ \frac{2\nu_B}{D_p} \left[\frac{y^2}{2} - \frac{p_B}{N_d} y \right] + C' \quad \dots \quad (11)$$

To evaluate C', we utilise the condition that at the collector base junction, the minority carrier concentration is negligible so that

$$y = 0$$

and
$$\frac{dy}{dx} = \left(\frac{dy}{dx} \right)_c \quad \text{say} \quad = -\frac{J_c}{eD_p N_d} \qquad \left. \begin{array}{c} \\ \\ \\ \end{array} \right\} \quad \dots \quad (12)$$

where J_c is the collector current density. From Eqs. (11) and (12), we have

$$C' = \left(-\frac{J_c}{eD_p N_d} \right)^2. \qquad \dots \quad (13)$$

Again, from Eq. (3), the emitter current density $J_e = J_p$ is given by

$$J_e = - eD_p \left(1 + \frac{p_e}{p_e + N_d} \right) \left(\frac{dp}{dx} \right)_e$$

$$= - eD_p N_d \left(\frac{1+2y_e}{1+y_e} \right) \left(\frac{dy}{dx} \right)_e, \qquad \dots \quad (14)$$

where the subscript e refers to the emitter base junction.

From Eqs. (11), (13) and (14), one obtains

$$\left(\frac{J_e}{eD_pN_d}\right)^2 = \frac{2}{D_p\tau_B}\left[\frac{2}{3}y_e + \left(\frac{1}{2} - \frac{p_B}{N_d}\right)y_e^2 - \frac{p_B}{N_d}y_e\right]$$

$$+\frac{2v_B}{D_p}\left[\frac{y_e^2}{2} - \frac{p_B}{N_d}y_e\right] + \left(\frac{J_c}{eD_pN_d}\right)^2, \qquad \ldots \ (15)$$

or
$$J_c = J_e\left[1 - \left(\frac{eD_pN_d}{J_e}\right)^2\frac{2}{D_p\tau_B}\left\{\frac{2}{3}y_e^3 + \left(\frac{1}{2} - \frac{p_B}{N_d}\right)y_e^2 - \frac{p_B}{N_d}y_e\right\} - \right.$$

$$\left.\left(\frac{eD_pN_d}{J_e}\right)^2\frac{2v_B}{D_p}\left\{\frac{y_c^2}{2} - \frac{p_B}{N_d}y_e\right\}\right]^{\frac{1}{2}}. \qquad \ldots \ (16)$$

Differentiating both sides of Eq. (16) with respect to J_e and noting that the transport factor β_{ce} for the grounded base mode is given by $\beta_{ce} = -\dfrac{dJ_c}{dJ_e}$, one obtains

$$\beta_{ce} = \frac{1 - \left[\frac{1}{J_e}\cdot\frac{(eD_pN_d)^2}{D_p\,\tau_B}\left\{2y_e^2 + \left(1-2\frac{p_B}{N_d}\right)y_e - \frac{p_B}{N_d}\right\} + \frac{1}{J_e}\frac{(eD_pN_d)^2v_B}{D_p}\right.}{\left[1 - \frac{2}{J_e^2}\frac{(eD_pN_d)^2}{D_p\tau_B}\left\{\frac{2}{3}y_e^3 + \left(\frac{1}{2} - \frac{p_B}{N_d}\right)y_e^2 - \frac{p_B}{N_d}y_e\right\}\right.}$$

$$\times \ \frac{\left.\left(y_e - \frac{p_B}{N_d}\right)\right]\frac{dy_e}{dJ_e}}{\left.\frac{2}{J_e^2}\frac{(eD_pN_d)^2v_B}{D_p}\left\{\frac{y_e^2}{2} - \frac{p_B}{N_d}y_e\right\}\right]^{\frac{1}{2}}} \qquad (17)$$

which is the general expression for β_{ce}.

4. RELATION BETWEEN THE CURRENT DENSITY AND INJECTED CARRIER CONCENTRATION

The relation for β_{ce} given above [Eq. (17)] has the drawback that it involves both J_e and y_e and a precise relationship between these two parameters is needed to obtain β_{ce} explicitly in terms of either J_e or y_e. Unfortunately, however, a rigorous relation cannot be obtained readily and some approximations have to be made keeping in view, as far as possible, the peculiar conditions which arise under high level condition of operation. Thus noting that under high level condition the diffusion constant D_p effectively increases to D'_p where $D'_p = D_p$

$\left(1 + \dfrac{y_e}{1 + y_e}\right)$, Webster (1954) obtained, for the case when recombination in the base region is negligible, the relation

$$J_e = \frac{eD_p N_d}{W}\left\{2y_e - ln\,(1 + y_e)\right\} = \frac{eD_p N_d P}{W} \qquad \ldots \ (18)$$

where $P = 2y$, $ln(1 + y_e)$ and W is the width of the base region of the transistor. In a practical transistor, however, some of the injected carriers are invariably lost by recombination in the base region and Eq.(18) needs some modification to take this into account. A simple method of doing this is as follows :

We recall that in the presence of recombination the expression for J_e at low injection level is approximately given by

$$J'_e = \frac{eD_p p_B}{W}\exp.\left(\frac{eV_e}{kT}\right)\frac{\dfrac{W}{(D_p\tau)^{\frac12}}}{\tanh\dfrac{W}{(D_p\tau)^{\frac12}}} \qquad (19)$$

and that in the absence of recombination by

$$J_e' = \frac{eD_p p_B}{W}\exp.\left(\frac{eV_e}{kT}\right), \qquad \ldots \ (20)$$

where J'_e is the emitter current density at low injection level and V_e the applied emitter to base d.c. potential. It is thus seen that the presence of recombination introduces a factor $\dfrac{W}{(D_p\tau)^{\frac12}}\Big/\tanh\dfrac{W}{(D_p\tau)^{\frac12}}$ in the expression for J'_e. We assume that to a first approximation this also holds for high injection level. Eq. (18) can then be modified as

$$J_e = \frac{eD_p N_d P}{W}\cdot\frac{\dfrac{W}{(D_p\tau)^{\frac12}}}{\tanh\dfrac{W}{(D_p\tau)^{\frac12}}} \qquad \ldots \ (21)$$

Next we argue that at high injection level the term τ appearing in Eq. (21) is a function of y as given by Eq. (6). From Eqs. (6) and (21) we then obtain

$$J_e = \frac{eD_p N_d P}{W}\cdot\frac{W\left(\dfrac{1 + y_e}{D_p\tau_B}\right)^{\frac12}}{\tanh W\left(\dfrac{1 + y_e}{D_p\tau_B}\right)^{\frac12}}, \qquad \ldots \ (22)$$

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when recombination occurs only in the volume and

$$J_e = \frac{eD_p N_d P}{W} \cdot \frac{W \left\{ \dfrac{\nu_B(1+y_e)}{D_p(1+2y_e)} \right\}^{\frac{1}{2}}}{\tanh\ W \left\{ \dfrac{\nu_B(1+y_e)}{D_p(1+2y_e)} \right\}^{\frac{1}{2}}}, \qquad \dots \ (23)$$

when recombination is confined only to the surface.

5. RECOMBINATION MECHANISM AND THE VALUE OF THE TRANSPORT FACTOR

An expression giving the value of the transport factor β_{ce} explicitly as a function of y_e can now be derived with the help of Eqs. (17), (22) and (23). It is, however, convenient at this stage to consider separately the effects of surface and volume recombinations. This procedure simplifies mathematical manipulation considerably and also helps to bring out clearly the role of the individual types of recombination processes on the operation of a transistor. Further, as will be shown presently, the results thus obtained are helpful in discussing the case when the two recombination processes are simultaneously operative. We consider first the case of surface recombination.

(i) *Recombination confined only to the surface.* When only surface recombination is operative, $\tau_B = \infty$. Keeping this in mind and substituting in Eq. (17) the value of J_e and $\dfrac{dy_e}{dJ_c}$ as obtained from Eq. (23), one obtains

$$\beta_{ee} = \frac{1 - \left(\tanh^2 \dfrac{W}{L'_{BS}} \right) \dfrac{y_e}{P} \left[1 - \dfrac{P}{(1+2y_e)^2} \left\{ \dfrac{1}{2} - \dfrac{\dfrac{W}{L_{BS}}}{\sinh \dfrac{2W}{L'_{BS}}} \right\} \right]^{-1}}{\left[1 - \left(\tanh^2 \dfrac{W}{L'_{BS}} \right) \dfrac{1+2y_e}{1+y_e} \cdot \dfrac{y_e^2}{P^2} \right]^{\frac{1}{2}}} \qquad \dots \ (24)$$

where
$$\frac{1}{L'_{BS}} = \left\{ \frac{\nu_B(1+y_e)}{D_p(1+2y_e)} \right\}^{\frac{1}{2}}. \qquad \dots \ (25)$$

For usual values of $\dfrac{W}{L'}$, the term

$$\left[1 - \frac{P}{(1+2y_e)^2} \left\{ \frac{1}{2} - \frac{\dfrac{W}{L'_{BS}}}{\sinh \dfrac{2W}{L'_{BS}}} \right\} \right]^{-1}$$

in the numerator of Eq. (24) is very nearly equal to unity. Eq. (24) can, there-
fore, be rewritten as

$$\beta_{ce} \approx \frac{1 - \dfrac{y_e}{P} \tanh^2 \dfrac{W}{L'_{BS}}}{\left[1 - \dfrac{y_e{}^2(1+2y_e)}{P^2(1+y_e)} \tanh^2 \dfrac{W}{L'_{BS}} \right]^{\frac{1}{2}}} \qquad \dots \ (26)$$

Eq. (26) predicts a tendency for β_{ce} to increase with $y_,$, approaching a limiting
value of

$$\beta_{ce} \approx 1 - \frac{W^2 v_B}{8 D_p} \qquad \dots \ (27)$$

for $y_e \gg 1$ and $\dfrac{W^2 v_B}{D_p} \ll 1$. For $y_e \ll 1$, Eq. (26) reduces to the form:

$$\beta_{ce} \approx 1 - \frac{W^2 v_B}{2 D_p}. \qquad \dots \ (28)$$

If the right hand side of Eq. (26) is expanded binomially, one obtains,
neglecting higher order terms,

$$\beta_{ce} \approx 1 - \left(\tanh^2 \frac{W}{L'_{S}} \right) \left[\frac{y_e}{P} - \frac{y_e{}^2 \left(\dfrac{1+2y_e}{1+y_e} \right)}{2P^2} \right]. \qquad \dots \ (29)$$

Using Eq. (29), the reciprocal of the transport factor β_{cl} for the grounded emitter
mode of operation can be obtained as

$$\beta_{eb}{}^{-1} = \left(\tanh^2 \frac{W}{L'_{BS}} \right) \left[\frac{y_e}{P} - \frac{y_e{}^2 \left(\dfrac{1+2y_e}{1+y_e} \right)}{2P^2} \right]$$

$$= \frac{W^2 v_B}{2 D_p} \left[\frac{\tanh^2 \dfrac{W}{L'_{BS}}}{\dfrac{W^2 v_B}{D_p}} \left\{ \frac{2y_e}{P} - \frac{y_e{}^2 \left(\dfrac{1+2y_e}{1+y_e} \right)}{P^2} \right\} \right]$$

$$= \frac{W^2 v_B}{2 D_p} \cdot k_1(z), \qquad \dots \ (30)$$

where

$$k_1(z) = \frac{\tanh^2 \dfrac{W}{L'_{BS}}}{\dfrac{W^2 \nu_B}{D_p}} \cdot \left[\frac{2y_e}{P} - \frac{y_e^2 \left(\dfrac{1 + 2y_e}{1 + y_e} \right)}{P^2} \right] \qquad \dots \quad (31)$$

It is easily shown that for $W \ll L'$

$$k_1(z) \approx \frac{2y_e(1 + y_e)}{P(1 + 2y_e)} - \left(\frac{y_e}{P} \right)^2 . \qquad \dots \quad (32)$$

$k_1(z)$ may be called the 'fall-off factor' and is analogous to the factors $g(z)$ and $m(z)$ of Webster (1954) and Hauri (1956) respectively. Like these latter factors, $k_1(z)$ tends to unity for $y_e \ll 1$ but for $y_e \gg 1$, the limiting value of this factor is 0.25 as compared to the value of 0.50 for both $g(z)$ and $m(z)$.

In the analysis presented above it has been assumed that L'_B is a function of y_e. Webster has, however, suggested that the surface recombination lifetime is independent of y_e and has ascribed the observed increase in the value of β_{ce} to the increase in the value of the effective diffusion constant. Accepting this point of view, the modified expression for β_{ce} is found to be

$$\beta_{ce} = \frac{1 - \dfrac{y_e}{P} \tanh^2 \dfrac{W}{L_{BS}}}{\left[1 - \dfrac{2\{y_e^2 - y_e + ln(1 + y_e)\}}{P^2} \tanh^2 \dfrac{W}{L_{BS}} \right]^{\frac{1}{2}}}, \qquad \dots \quad (33)$$

where
$$\frac{1}{L_{BS}} = \left(\frac{\nu_B}{D_p} \right)^{\frac{1}{2}} .$$

For low values of $y_e (y_e \ll 1)$, Eq. (33) reduces to the form of Eq. (28) and for high values of $y_e (y_e \gg 1)$, Eq. (33) simplifies to

$$\beta_{ce} \approx 1 - \frac{W^2 \nu_B}{4D_p} \qquad \dots \quad (34)$$

which shows that the loss due to surface recombination is halved at high current densities. This is in accordance with the conclusions arrived at by both Webster and Hauri. For intermediate values of y_e, however, the nature of variation of

this term can be studied by considering the variation of the 'fall-off factor' which for this case may be written as

$$k_2(z) = \frac{\tanh^2 \dfrac{W}{L_{BS}}}{\left(\dfrac{W}{L_{BS}}\right)^2} \left[\frac{2y_e}{P} - \frac{2\{y_e^2 - y_e + ln(1 + y_e)\}}{P^2} \right] \qquad \ldots \quad (35)$$

$$\approx \frac{2y_e}{P} - \frac{2\{y_e^2 - y_e + ln(1 + y_e)\}}{P^2} \qquad \ldots \quad (36)$$

It is easily seen from Eq.(36) that the limiting values of the factor $k_2(z)$ for $y_e \ll 1$ and $y_e \gg 1$ are 1 and 0.5 respectively in agreement with those of $g(z)$ and $m(z)$. For intermediate values of y_e, $k_2(z)$ differs slightly from both. Table I compares these values.

TABLE I

Values of the factors $g(z)$, $m(z)$, $k_1(z)$ and $k_2(z)$:

y_e	$g(z)$	$m(z)$	$k_1(z)$	$k_2(z)$
1	2	3	4	5
0.01	0.99	0.99	0.97	1.00
0.05	0.95	0.98	0.89	0.96
0.10	0.92	0.96	0.80	0.94
0.50	0.75	0.86	0.57	0.80
1.00	0.67	0.79	0.44	0.72
2.00	0.60	0.72	0.34	0.64
5.00	0.55	0.64	0.29	0.57
10.00	0.52	0.60	0.26	0.54

(ii) *Recombination confined only to the volume.* When the volume recombination term in Eq.(17) plays the dominant role (i.e. $v_B = 0$). one obtains with the help of Eq. (22) the following expression for β_{ee}

$$\beta_{ce} = \frac{1 - \left(\tanh^2 \dfrac{W}{L_{BV'}}\right) \dfrac{y_e}{P} \left[1 - \dfrac{P}{1 + 2y_e}\left(\dfrac{1}{2} - \dfrac{\overline{L_{BV'}}}{\sinh \dfrac{2W}{L_{BV'}}} \right) \right]^{-1}}{\left[1 - \left(\tanh^2 \dfrac{W}{L_{BV'}}\right) \dfrac{y_e^2}{P^2}\left(1 + \dfrac{4}{\varepsilon}y_e\right) \right]^{\frac{1}{2}}}, \qquad (37)$$

where
$$\frac{1}{L_{BV}'} = \left(\frac{1 + y_e}{D_p \tau_B} \right)^{\frac{1}{2}} \qquad \qquad \qquad \dots (38)$$

On making the same assumption as was made in deriving Eq. (26), Eq. (37) can be simplified to

$$\beta_{ce} \approx \frac{1 - \dfrac{y_e}{P} \tanh^2 \dfrac{W}{L_{BV}'}}{\left[1 - \left(\dfrac{y_e}{P} \right)^2 \left(1 + \dfrac{4}{3} y_e \right) \tanh^2 \dfrac{W}{L_{BV'}} \right]^{\frac{1}{2}}} \qquad \dots (39)$$

From Eq (39) it is easily seen that for $y_e \ll 1$, the expression for β_{ce} reduces to

$$\beta_{ce} \approx 1 - \frac{1}{2} \tanh^2 \frac{W}{D_p \tau_B} \approx 1 - \frac{W^2}{2 D_p \tau_B} \qquad \qquad \dots (40)$$

which is the usual expression for low level condition of operation. When $y_e \gg 1$, the expression for β_{ce} takes the form

$$\beta_{ce} \approx 1 - \frac{1}{3} \left(\tanh^2 \frac{W}{D_p \tau_B} \right) y_e, \qquad \qquad \dots (41)$$

provided $\dfrac{W^2}{D_p \tau_B} \cdot y_e \ll 1$.

Assuming $\dfrac{W}{L_{BV'}} \ll 1$, one can obtain from Eq. (39) and approximate expression for β_{cb}^{-1} of the form

$$\beta_{cb}^{-1} = \frac{W^2}{D_p \tau_B} \left[\frac{y_e}{P} (1 + y_e) \frac{y_e^2}{2P^2} \left(1 + \frac{4}{3} y_e \right) \right] \qquad \dots (42)$$

It may be noted here that Matz (1958) has suggested an expression for β_{cb} in the form of an integral which when evaluated gives a relation similar to Eq. (42).

From Eq. (42) an expression for the 'fall off factor' for this case can be written down directly. Denoting this by the term $k_3(z)$, we have

$$k_3(z) = \frac{2y_e(1 + y_e)}{P} - \frac{y_e^2}{P^2} \left(1 + \frac{4}{3} y_e \right) . \qquad \dots (43)$$

It is to be noted that $k_3(z)$, unlike the factors $k_1(z)$ and $k_2(z)$, increases with y_e which shows that the transport factor β_{cb} actually decreases with increasing y_e. Thus the term 'fall off factor' is a misnomer for this case but has been retained for the sake of uniformity. From Eq. (43) it follows easily that $k_3(z)$ is equal

to 1 for $y_e \ll 1$ and is equal to $\frac{2}{3} y_e$ for $y_e \gg 1$. Its values in the range $.01 < y_e < 10$ are given in Table II.

TABLE II

Values of the factors $k_3(z)$, $g'(z)$ and $m'(z)$.

y_e	$k_3(z)$	$g'(z)$	$m'(z)$
1	2	3	4
0.01	1.01	1.01	1.00
0.05	1.03	1.05	1.01
0.10	1.07	1.10	1.02
0.50	1.35	1.59	1.11
1.00	1.67	2.31	1.21
2.00	2.39	3.90	1.41
5.00	4.47	9.21	2.01
10.00	7.87	18.60	3.00

'Fall off factors' for the cases considered by Webster (1954) and Hauri (1956) may be derived easily by examining the expressions for β_{cb}^{-1} given by these authors and are found to be

$$\left.\begin{array}{l} g'(z) = 1 + P \quad \text{[Webster]} \\ m'(z) = \{m(z)\}\{1 + \frac{2}{3} m(z) y_e\} \quad \text{[Hauri]} \end{array}\right\} \quad \dots \quad (44)$$

Values of $g'(z)$ and $m'(z)$ as given by Eq. (44) for different values of y_e are given in columns 3 and 4 of Table II.

It should be noted however that the type of volume recombination discussed above is the bimolecular one. The other type of interest is monomolecular volume recombination in which the lifetime tends to become independent of y_e. A solution for this case may be obtained in the manner outlined above yielding for β_{ce} the expression

$$\beta_{ce} = \frac{1 - \frac{y_e}{P} \tanh^2 \frac{W}{L_{BV}}}{\left[1 - \frac{2\{y_e{}^2 - y_e + ln(1 + y_e)\}}{P^2} \tanh^2 \frac{W}{L_{BV}}\right]^{\frac{1}{2}}}, \quad \dots \quad (45)$$

where $\qquad L_{BV} = (D_p \tau_B)^{\frac{1}{2}}$.

For the limiting case $y_e \gg 1$, Eq. (45) reduces to the form

$$\beta_{ce} \approx 1 - \frac{W^2}{4 D_p \tau_B}; \quad \dots \quad (46)$$

showing that β_{ce}, after an initial increase with y_e, attains a limiting value. This is in agreement with the conclusions arrived at by both Webster (1954) and Hauri (1956). The increase of β_{ce} with y_e as is observed here is due to the apparent increase of the diffusion constant D_p.

As regards the 'fall off factor' for this case, the formal resemblance between Eq. (45) and Eq. (33) evidently suggests that this factor should be identical with the factor $k_2(z)$. A similar argument would show that the 'fall off factors' for the cases treated by Webster and Hauri are identical with the factors $g(z)$ and $m(z)$ respectively.

(iii) *Surface and volume recombination operative simultaneously.* So far the two types of recombination processes have been considered separately for reasons already mentioned. The concept of the "fall off factor", however, enables one to infer the law of variation of β_{ce} with y_e when both the processes are simultaneously operative. We recall (Webster 1954) that at low injection level the expression for β_{cb}^{-1} is given as the sum of three terms corresponding to the contributions of surface recombination, volume recombination and emitter efficiency. Assuming that the emitter efficiency is unity, the expression for β_{cb}^{-1} can be written as

$$\beta_{cb}^{-1} = a . \left\{ \begin{array}{c} k_1(z) \\ k_2(z) \end{array} \right\} + b . \left\{ \begin{array}{c} k_2(z) \\ k_3(z) \end{array} \right\}, \qquad \qquad \ldots \text{ (47)}$$

where a and b are constants depending on the low level values of the surface recombination term and the volume recombination term respectively. The terms within the parentheses in Eq. (47) take into account the possible alternative modes of recombination mentioned earlier.

6. EFFECT OF EMITTER EFFICIENCY ON THE TRANSPORT FACTOR

The analysis given in the preceding section has the drawback that it is based on the assumption $\gamma = 1$, i.e. $J_n = 0$. In a practical transistor, however, this is not so and the expression for J_p as given by Eq. (3) will consequently be inadequate. When $J_n \neq 0$, one obtains from Eqs. (1) and (2)

$$E = \frac{J_n - ebD_p \dfrac{dp}{dx}}{e\mu_n(p + N_d)}, \qquad \qquad \ldots \text{ (48)}$$

and $\qquad J_p = - eD_p \left(1 + \dfrac{p}{N_d} \right) \left\{ 1 - \dfrac{p}{b} \dfrac{J_n}{J_p(p+N_d)} \right\}^{-1} \dfrac{dp}{dx} \qquad \ldots \text{ (49)}$

where $\qquad\qquad\qquad b = \dfrac{\mu_n}{\mu_p}$

From Eqs. (3) and (49), it is seen that the effective value of the diffusion constant in the latter case is larger than the value obtained with $J_n = 0$. This will make the value of β larger. In actual practice, however, this increase is not appreciable. Thus even if J_n becomes high enough to make $\gamma = 0.9$, the percentage increase in D_p' is not more than 5. The assumption $J_n = 0$, therefore, does not affect appreciably the results reported in Secs. 3—5.

7. DISCUSSION

To summarise the results obtained in the preceding sections we note that the values of the 'fall-off factors' $g(z)$, $m(z)$ and $k_2(z)$ in Table I are in reasonable agreement while those of the factor $k_1(z)$ are much smaller in magnitude for $y_e \gg 1$. This discrepancy arises because of the fact that in deriving Eq. (26), $L_{R}s'$ was assumed to be a function of y_e, its values increasing with increasing values of y_e whereas the other factors, viz. $g(z)$, $m(z)$ and $k_2(z)$, were not based on such assumption. A glance at Table I would show that an experimental test of the validity or otherwise of the aforesaid assumption is best made by measuring β at high level of operation. This is, however, difficult because of the increased importance of the emitter efficiency term and volume recombination at high level and of the lack of an accurate method of independent measurement of γ (Deb and Daw, 1958). In view of this we can make a check only for the low levels of operation even though at such levels the values of β as predicted by the different theoretical relations in Table I do not differ widely from each other. Let us consider the case when y_e is increased from 0 to 0.1. Table I would show that for this change in y_e the percentage increase in β as predicted by the fall off factors $g(z)$, $m(z)$. $k_2(z)$ and $k_1(z)$ are 8, 4, 6 and 20 respectively. Experimentally measured values of change in current amplification factor as obtained for transistor types 0C70 and 0C602 for the same conditions are 14% for the former and 8.5% for the latter. Bearing in mind the fact that the effects of the volume recombination and emitter efficiency teerms would be to make the experimentally measured percentage change in current amplification factor lower than the true increase in the transport factor β it would appear that the assumption regarding the dependance of surface recombination lifetime on y_e is somewhat justified and that the factor $k_1(z)$ gives the more correct value of fall-off factor than any of the other three.

Taking the case of volume recombination we find from Table II that the values of the factor $k_3(z)$ agree quite well with those of $g'(z)$ for $y_e \leqslant 0.1$ At higher values of y_e, however, the increase of $g'(z)$ is faster than that of $k_3(z)$. This is understood if it is recalled that while in the present analysis the apparent increase in the diffusion constant of the minority carriers with the level of injection has been taken account of, Webster in his treatment had ignored it. As regards $m'(z)$, this factor has the slowest variation with y_e and consequently gives the highest value of β_{ce}. This is because the assumed nature of bimolecular

recombination in this case is different from that given by Eq. (6). Thus in the limiting case $y_e >> 1$, the approximate expressions for β_c, as given by Webster, Hauri and Eq. (41) are as follows :

$$\beta_{c2} \approx 1 - \frac{W^2}{2D_p\tau_B} y_e \qquad \text{(Webster)} \qquad \dots \quad (50)$$

$$\beta_{c3} \approx 1 - \frac{W^2}{12D_p\tau_B} y_e \qquad \text{(Hauri)} \qquad \dots \quad (51)$$

$$\beta_{ee} \approx 1 - \frac{W^2}{3D_p\tau_B} y_e \qquad \text{(obtained from Eq. 41)} \quad \dots \quad (52)$$

Here again a categorical experimental verdict in favour of the one or the other of these equations is difficult because of the increasing role of the emitter efficiency term γ under the limiting condition $y_e >> 1$.

It is, however, to be noted that some earlier experimental results (Evans 1956, Deb and Daw 1958) indicate that the effective lifetime passes through a maximum as y_e is increased from a low value. Eq. (6) used in the present analysis is the only relation suggested so far which can account for such a variation. As such Eq. (47) undoubtedly deserves careful consideration. Assuming now that Eq. (6) is valid for a practical transistor, one obtains from Eqs. (32), (43) and (47) the following expression for β_{cb}^{-1}.

$$\beta_{cb}^{-1} = a \left[\frac{2y_e(1+y_e)}{P(1+2y_e)} - \frac{y_e^2}{P^2} \right] - b \left[\frac{2y_e}{P}(1+y_e) - \frac{y_e^2}{P^2} \left(1 + \frac{4}{3} y_e\right) \right] \quad (53)$$

where $\quad a = \dfrac{W^2 \nu_B}{2D_p} \quad$ and $\quad b = \dfrac{W^2}{2D_p\tau}.$ $\qquad \dots \quad (54)$

A rough check on the validity of Eq. (53) may be made as follows. The values of τ_B and ν_B for types OC70 and OC602 transistors determined earlier (Deb and Daw, 1958) are given in columns 2 and 3 of Table III. In column 4 are given values of β_{ce} obtained from these values and Eqs. (53) and (54) for $y_e = 1$. Column 5 gives the experimental values of the current amplification factor α and column those of γ using the results of the preceding two columns. In the last column are given the values of $\sigma_e L_e$ obtained from the relation

$$\frac{\sigma_b W}{\sigma_e L_e} = \frac{1 - \gamma}{\gamma} \qquad \dots \quad (55)$$

These results are in reasonable agreement with the expected values (Early 1953, Webster 1954 and Kaufmann 1959) and lend support to the validity of Eq. (53)

TABLE III

Values of $\sigma_e L_e$ at $y_e = 1$

Transistor type	τ_B μsec.	$1/\nu_B$ μsec.	β_{ce}	α	γ	$\sigma_e L_e$ mhos
1	2	3	4	5	6	7
OC70	86.4	17.2	0.979	0.973	0.994	0.67
OC602	51.8	26.4	0.979	0.977	0.999	1.60

With regard to the role of the term γ it is found that this does not affect sensibly the utility of Eq. (17). It is interesting to note that the presence of an appreciable electronic component of current—apart from decreasing β through a decreased γ value—also increases the same through an increased diffusion constant. In practice, however, the decrease in γ masks completely any increase arising out of this last effect.

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FREQUENCY OF THE THREE-PHASE R-C COUPLED OSCILLATOR PART II.
INDUCTIVE ANODE LOAD RESISTANCE

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ABSTRACT. When the three stages of a conventional three phase R–C oscillator are identical, the oscillations normally produced are of radio frequency $\omega = \sqrt{3}/CR$, where R and C are tuning resistance and capacitance. It was shown in Part I (Rakshit and Mallik, 1955) that this simple formula holds when the load resistance is non-reactive and the cathode impedance is zero or resistive. The case of inductive anode resistance when cathode impedance is (i) resistive and (ii) reactive has been discussed in the present paper. For resistive cathode impedance an inductance in series with anode resistance R causes an increase in frequency over the $\sqrt{3}/CR$ value up to a certain magnitude of the inductance but for still higher magnitudes the frequency goes on decreasing.

The effect of reactive cathode when the anode load is inductive is the same as that for non-reactive anode load resistance discussed in Part I. Inductive cathode impedance decreases the frequency of oscillation from the value for resistive cathode impedance while capacitive cathode impedance generally increases the same.

INTRODUCTION

It has been shown in Part I that the frequency of the three phase *R-C* oscillator is dependent on the cathode circuit impedance. The simple expression for frequency $\omega = \sqrt{3}/CR$ holds good, only when the cathode circuit impedance is zero or purely resistive and anode load resistance is non-reactive. When the cathode impedance is reactive the generated frequency will be greater or less than $\sqrt{3}/CR$ depending upon whether the cathode impedance is capacitive or inductive. The effect of cathode impedance is prominent when the generated frequency is much below or much above the cathode resonant frequency.

In the present paper we shall discuss the condition when cathode impedance is zero, resistive, or reactive but anode load resistance is inductive. The addition of inductance with the load resistance increases the load impedance and makes it possible to maintain oscillation at comparatively low values of load resistance. In fact in our attempt to generate very high frequency it has been found that these inductances are essential for maintenance of oscillation (Rakshit and Mallik, 1953). Without the small inductance the oscillation frequency $\omega =$

$\sqrt{3}/CR$ and the phase shift per stage is 120 degrees. The presence of the small inductance causes a decrease in phase shift at this frequency and as a result the frequency of the maintained oscillation is increased. This increase in frequency with increasing inductance continues up to a certain maximum value and thereafter the frequency goes on decreasing and finally for still larger values of inductance the frequency becomes less than $\sqrt{3}/CR$ value.

THEORETICAL CONSIDERATION

In this case the anode load Z_0 consists of the tuning capacitor C shunted by the series combination of R and its associated inductance L.
Hence,

$$Z_0 = \frac{[R + j\omega\{L(1-\omega^2 LC) - CR^2\}]}{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2} \qquad \dots \ (1)$$

The phase angle of the load is given by

$$\tan\theta = \frac{\omega\{L(1-\omega^2 LC) - CR^2\}}{R}$$

and the frequency of the radio frequency oscillation is accordingly given by $\tan\theta = -\sqrt{3}$ (Rakshit and Bhattacharyya, 1946)

Or
$$\omega^3 L^2 C + \omega(CR^2 - L) - \sqrt{3}R = 0 \qquad \dots \ (2)$$

This is a cubic equation with one real root, the other two being imaginary. The real value of ω is given by

$$\omega = [P + (P^2 + Q^3)^{\frac{1}{2}}]^{1/3} + [P - (P^2 + Q^3)^{\frac{1}{2}}]^{1/3} \qquad \dots \ (3)$$

where

$$P = \frac{\sqrt{3}R}{2L^2C} \qquad \text{and} \qquad Q = \frac{CR^2 - L}{3L^2C}$$

For convenience of numerical computation, this may be written in the following form by putting x for CR^2/L. We then have

$$\omega = \frac{x^{2/3}}{CR}\left[\left\{\frac{\sqrt{3}}{2} + \left(0.75 + \frac{(x-1)^3}{27x}\right)^{\frac{1}{2}}\right\}^{1/3}\right.$$

$$\left. + \left\{\frac{\sqrt{3}}{2} - \left(0.75 + \frac{(x-1)^3}{27x}\right)^{\frac{1}{2}}\right\}^{1/3}\right] \qquad \dots \ (3a)$$

$$= \frac{K}{CR}, \qquad \dots \ (3b)$$

where $K = x^{2/3}\left[\left\{\dfrac{\sqrt{3}}{2} + \left(0.75 + \dfrac{(x-1)^3}{27x}\right)^{\frac{1}{2}}\right\}^{1/3}\right.$

$$+ \left.\left\{\dfrac{\sqrt{3}}{2} - \left(0.75 + \dfrac{(x-1)^3}{27x}\right)^{\frac{1}{2}}\right\}^{1/3}\right].$$

It is obvious that if a table giving K for different values of x according to this equation is available we can at once calculate ω from Eqn. (3b) by first finding x for the particular combination of R, L and C. For this purpose the calculated values of K for different values of x ranging from 0.01 to 100 have been given in Table I. The results are also plotted in Fig. 1. For values of $x > 100$ or $<$
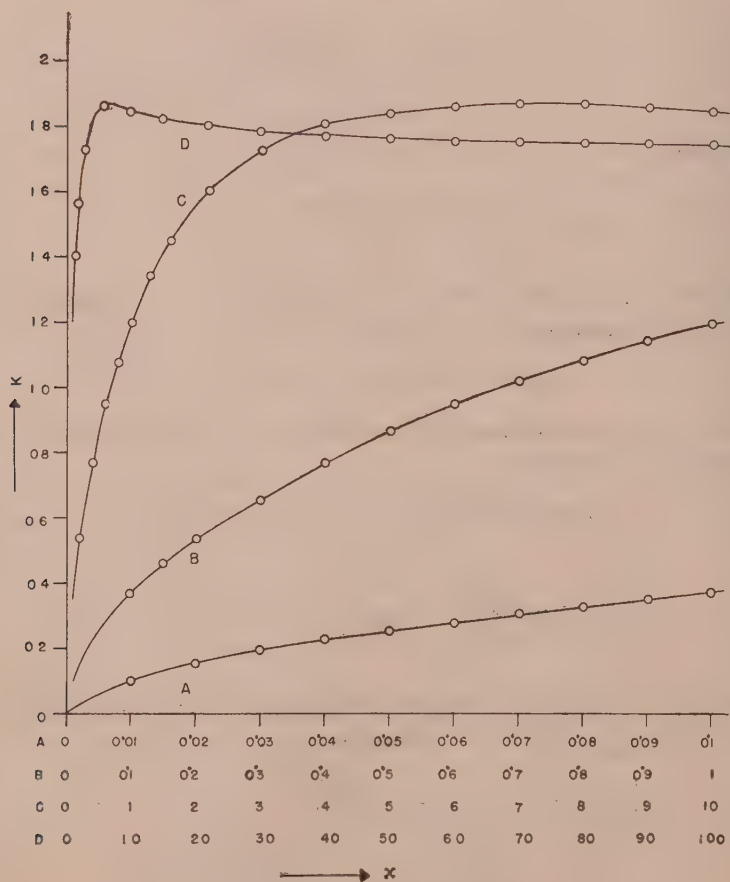
Fig. 1. Variation of K with x.

<0.01 we can calculate the value of ω after making some approximations leading to Eqns. (4a) and (7a). The error involved in these approximations is quite

small.   Thus for $x = 100$ if ω is calculated by Eqn. (4a) instead of Eqn. (3a), the result is 0.117% greater than the exact value. For larger values of x the error due to approximation will gradually diminish. Similarly when $x = 0.01$ if ω is calculated by Eqn. (7a) the result is 1.2% greater than the exact value. For still smaller values of x the error due to approximation is still less.

TABLE I
x vs. K

x	K	x	K	x	K
0.01	0.1073	1	1.2010	6	1.8633
0.02	0.1547	1.1	1.2486	7	1.8661
0.03	0.1930	1.2	1.2971	8	1.8637
0.04	0.2251	1.3	1.3389	9	1.8551
0.05	0.2540	1.4	1.3789	10	1.8537
0.06	0.2805	1.5	1.4154	11	1.8479
0.07	0.3049	1.6	1.4492	12	1.8416
0.08	0.3272	1.7	1.4804	13	1.8363
0.09	0.3495	1.8	1.5134	15	1.8263
0.1	0.3699	1.9	1.5363	22	1.8014
0.15	0.4606	2	1.5613	25	1.7945
0.20	0.5375	2.2	1.6062	30	1.7854
0.30	0.6556	2.5	1.6622	40	1.7735
0.40	0.7724	2.8	1.7108	50	1.7648
0.50	0.8660	3	1.7320	60	1.7559
0.60	0.9477	3.4	1.7718	70	1.7547
0.70	1.0217	3.7	1.7949	80	1.7534
0.80	1.0859	4	1.8137	90	1.7520
0.90	1.1445	5	1.8488	100	1.7472

It will be noted that when Q is negative and $P^2 < |Q^3|$, $\{P^2+Q^3\}^{\frac{1}{2}}$ is imaginary and Eqn. (3) can be solved with the help of trigonometry but not algebraically.

The expression for ω as given in Eqn.(3) can be reduced to a simpler approximate form when one of the terms under the square root becomes small in comparison with the other. Thus when $CR^2 > L$, i.e. Q is positive and $P^2 << Q^3$ i.e. $\dfrac{81CR^2}{4L} << \left(\dfrac{CR^2}{L}-1\right)^3$, we get (see Appendix I)

$$\omega = \frac{\sqrt{3}R}{CR^2 - L}\left[1 - \frac{3R^2L^2C}{(CR^2-L)^3}\right] \qquad \dots \ (4)$$

Or

$$\omega = \frac{\sqrt{3x}}{CR(x-1)}\left[1 - \frac{3x}{(x-1)^3}\right] \qquad \dots \ (4a)$$

As a further approximation when $CR^2 >> L$ we may neglect the term $\dfrac{3R^2L^2C}{(CR^2-L)^3}$ and Eqn. (4) reduces to

$$\omega = \frac{\sqrt{3}R}{CR^2-L} \qquad \qquad \dots \text{(5)}$$

This shows that when L is so small that $P^2 << Q^3$ the small inductance associated with the anode load resistance causes an increase in frequency. In the limit when $L = 0$, we get the simple expression. $\omega = \sqrt{3}/CR$

When Q is negative and $P^2 < |Q^3|$, solving Eqn. (3) trigonometrically gives

$$\omega = r^{1/3}\; 2\cos\frac{\phi}{3} \qquad \qquad \dots \text{(6)}$$

where $r = |Q^{3/2}|$ and $\cos\phi = P/Q^{3/2}$. Substituting the values of P and Q,

$$\omega = \left|\frac{1}{3LC}\left(\frac{CR^2}{L}-1\right)\right|^{\frac{1}{2}} 2\cos\frac{\cos^{-1}\left|\dfrac{81CR^2}{4L\left(\dfrac{CR^2}{L}-1\right)^3}\right|^{\frac{1}{2}}}{3} \qquad \dots \text{(6a)}$$

When R is very small Eqn. (6) reduces to (see Appendix II)

$$\omega = \frac{1}{\sqrt{LC}} + \frac{\sqrt{3}R}{2L} \qquad \qquad \dots \text{(7)}$$

Or in terms of x,

$$\omega = \frac{\sqrt{x}}{CR}\left(1 + \frac{\sqrt{3x}}{2}\right) = \frac{1}{\sqrt{LC}}\left(1 + \frac{\sqrt{3x}}{2}\right) \qquad \dots \text{(7a)}$$

This shows that when R is of the order of 0 the frequency generated by a 3-phase oscillator is approximately equal to the frequency generated by au orthodox $L-C$ oscillator with same circuit parameters but always greater than $1/\sqrt{LC}$. The expression $\omega = 1/\sqrt{LC}$ in case of 3-phase oscillator can also be obtained by putting $R = 0$ in Eqn. (2) but from the expression for $\tan\theta$ it is obvious that oscillations cannot be maintained when $R = 0$.

The above Eqn. (6) holds good in the case $CR^2 < L$ upto the limit $P^2 = |Q^3|$. Under this limiting condition $\cos\phi = 1$ and hence $\cos\dfrac{\phi}{3} = 1$ and

$$\omega = 2\left\{\frac{\sqrt{3}R}{2L^2C}\right\}^{1/3} \qquad \qquad \dots \text{(8)}$$

It will be noted that when the above condition holds, $P^2 + Q^3 = 0$ and the same expression for ω is also obtained directly from Eqn. (3).

When, on the other hand, $P^2 >> Q^3$

$[P + (P^2 + Q^3)^{\frac{1}{2}}]^{1/3}$ is of the order of $(2P)^{1/3} + \dfrac{1}{12} \cdot \dfrac{Q^3}{P^2} \ (2P)^{1/3}$ and $[P - (P^2 + Q^3)^{\frac{1}{2}}]^{1/3}$

is of the order of $-\left(\dfrac{Q}{2P} \right)^{1/3}$.

$$\therefore \quad \omega = (2P)^{1/3} - \left(\frac{Q}{2P} \right)^{1/3} + \frac{1}{12} \ \frac{Q^3}{P^2} \ (2P)^{1/3} \qquad \qquad \dots \ (9)$$

Or, in terms of x,

$$\omega = \frac{1}{CR} \left[\ (\sqrt{3x^2})^{1/3} - \frac{x^{1/3}(x-1)}{3^{7/6}} + \frac{(x-1)^3}{x^{1/3}3^{29/6}} \right] \qquad \dots \ (9a)$$

For generating very high frequency the magnitudes of R, C and L should be very small; $(CR^2 - L)$ is then usually a very small quantity. In the special case when $CR^2 - L = 0$ i.e. $x = 1$, Eqn. (9a) reduces to

$$\omega = \frac{3^{1/6}}{CR} = \frac{1.201}{CR} \qquad \qquad \dots \ (10)$$

Condition for Maintenance of Oscillation :

We have so far considered the phase shift per stage necessary to produce oscillations. For maintenance of these oscillations the gain A per stage must at least be unity. If anode load impedance of each oscillator stage is Z_0 as given in Eqn. (1), this means

$$g_m Z_0 = A \geqslant 1,$$

where g_m is the mutual conductance of the oscillator valves. To evaluate Z_0 from Eqn. (1) in terms of circuit parameters we have to substitute the value of ω in terms of these parameters. Taking ω as given by Eqn. (3b), we find

$$Z_0 = \frac{R \left[1 + \dfrac{K^2}{x^2} \left\{ 1 - \dfrac{K^2}{x} - x \right\}^2 \right]^{\frac{1}{2}}}{K^2 + \left(\dfrac{K^2}{x} - 1 \right)^2}$$

For a particular combination of C, R and L, since x and K are pure numbers Z_0 is obtained in terms of R only. It is easy to see that Z_0 can likewise be expressed in terms of any one of the circuit parameters. The magnitude of Z_0 in terms of R for various values of x and the condition that has to be satisfied to maintain oscillation in those cases are given in Table II and plotted in figure 2. It may be noted that when $L = 0$ we have the simple $R - C$ oscillator and $\omega = \sqrt{3}/CR$,
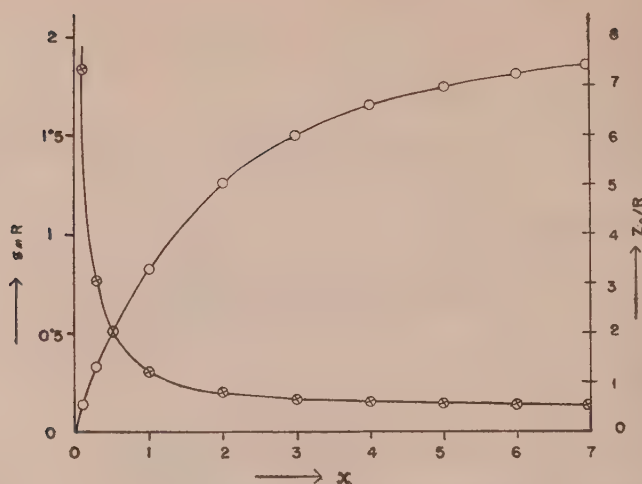
Fig. 2. Z_0 in terms of R, and g_m R required for maintenance of oscillation, for different values of x.

TABLE II

Z_0 *vs.* x and minimum g_m R for oscillation

x	Z_0	Min. g_mR	Remarks
∞	0.5 R	2	Purely resistive load
6.964	0.53 R	1.866	$\dfrac{\partial \omega}{\partial L} = 0$
6	0.546 R	1.813	
5	0.568 R	1.759	
4	0.603 R	1.659	
3	0.666 R	1.50	$\omega_{RC} = \omega_{RLC}$
2	0.792 R	1.263	
1	1.221 R	0.818	$CR^2 = L$
0.5	2 R	0.5	$\dfrac{\partial \omega}{\partial R} = 0$
0.3	3.06 R	0.327	
0.1	7.342 R	0.136	

$Z_0 = R/2$ and condition for maintenance is $g_m R \geqslant 2$. When $R = 0$, ω is of the order of $1/\sqrt{LC}$ (Eqn. 7), Z_0 is very large and oscillation can be maintained with only a very small value of g_m.

NATURE OF VARIATION OF ω WITH THE TUNING PARAMETERS IN THE THREE–PHASE OSCILLATOR

The equations deduced above give the variations of ω when the tuning parameters are changed. Some important results are obtained if we study the variation with any one parameter at a time.

(a) *Variation of ω with R.*

1. In the case of the $R-C$ oscillator the frequency is given by $\omega = \sqrt{3}/CR$.

$$\frac{\partial \omega}{\partial R} = -\frac{\sqrt{3}}{CR^2} \qquad \qquad \dots \ (11)$$

and is always negative.

2. In the case of the $R-L-C$ oscillator the real value of ω is given by Eqn. (3), but it is rather difficult to find $\partial\omega/\partial R$ from this expression for ω. It is more convenient to find $\partial\omega/\partial R$ starting from Eqn. (2). Thus if we put

$$\omega^3 L^2 C + \omega(CR^2 - L) - \sqrt{3}R = F$$

then

$$\frac{\partial \omega}{\partial R} = -\frac{\partial F}{\partial R} \bigg/ \frac{\partial F}{\partial \omega} = -\frac{2\omega CR - \sqrt{3}}{3\omega^2 L^2 C + (CR^2 - L)} \qquad \dots \ (12)$$

For any particular set of values for R, L and C, we find ω from Eqn. (3) and substituting this value of ω in Eqn. (12) we get $\partial\omega/\partial R$. Now from Eqn. (2), $\omega^2 L^2 C + (CR^2 - L) = \dfrac{\sqrt{3}R}{\omega}$ and hence Eqn. (12) may be written as

$$\frac{\partial \omega}{\partial R} = \frac{\sqrt{3} - 2\omega CR}{2\omega^2 L^2 C + \dfrac{\sqrt{3}R}{\omega}} \qquad \qquad \dots \ (12a)$$

The denominator being always positive, we see that the nature of variation of ω is dependent upon whether $2\omega CR$ is greater or less than $\sqrt{3}$. In the limit when $L = 0$, we have the simple $R-C$ oscillator and $\omega = \sqrt{3}/CR$ and Eqn. (12a) gives

$$\frac{\partial \omega}{\partial R} = -\frac{\sqrt{3}}{\sqrt{3}R/\omega} = -\frac{\omega}{R} = -\frac{\sqrt{3}}{CR^2}$$

exactly as in Eqn. (11a). Equation (12a) further shows that if the set of values of R, L and C is such that $\sqrt{3}/2CR > \omega$, $\partial\omega/\partial R$ is positive for the $R-L-C$ oscillator and not negative as is always the case with simple $R-C$ oscillator. In fact for very small values of R, $\partial\omega/\partial R$ is always positive, since $2CR\omega$ approaches zero as R becomes negligibly small.

Further if R is gradually increased, keeping the above magnitudes of L and C constant, we come to a value of $\omega = \sqrt{3}/2CR$ for which $\partial\omega/\partial R = 0$. This maximum value of ω for which $\partial\omega/\partial R = 0$ may be calculated as follows :
We have $\omega_{max} = \sqrt{3}/2CR_{max}$, where R_{max} is the value of R for which $\partial\omega/\partial R = 0$. Substituting this value of R in Eqn. (2), we have

$$\omega^3 L^2 C - \frac{3}{4\omega C} - \omega L = 0$$

or $\qquad\qquad\qquad 4\omega^4 L^2 C^2 - 4\omega^2 LC - 3 = 0$

Solving as a quadratic in $2\ \omega^2 LC$, we get

$$2\omega^2 LC = 3 \text{ or } -1,$$

the negative value being obviously inadmissible.

$$\therefore \qquad \omega_{max} = \left[\frac{3}{2LC} \right]^{\frac{1}{2}} \qquad\qquad \dots \text{(13a)}$$

and $\qquad\qquad R_{max} = \left[\frac{L}{2C} \right]^{\frac{1}{2}} \qquad\qquad \dots \text{(13b)}$

These equations show that if the ratio L/C is kept constant, the maximum frequency for which $\partial\omega/\partial R = 0$ will correspond to a fixed value of resistance irrespective of the individual magnitudes of L and C. The magnitude of the maximum frequency which depends on the product LC will however be different for different magnitudes of L and C.

For further increase in R, $\delta\omega/\partial R$ is negative and at a certain value of R the frequency of the $R-L-C$ oscillator once again becomes equal to $\sqrt{3}/CR$ as if L were not present. This value of R for which the presence or absence of L makes no difference in the frequency of the oscillations may be obtained by putting $\omega = \sqrt{3}/CR$ in Eqn. (2). This gives

$$\frac{3\sqrt{3}L^2 C}{C^3 R^3} + \frac{\sqrt{3}CR^2}{CR} - \frac{\sqrt{3}L}{CR} - \sqrt{3}R = 0$$

or $\qquad\qquad\qquad \frac{\sqrt{3}L}{CR}\left(\frac{3L}{CR^2} - 1 \right) = 0$

which is satisfied by

$\qquad\qquad$ (i) $L = 0$ and (ii) $CR^2 = 3L$ $\qquad\qquad \dots \text{(14)}$

Condition (i) is obvious and condition (ii) gives the desired value of R.

It is interesting to compare the value of $\partial\omega/\partial R$ at the same value of ω, for the two cases —(i) $R-C$ oscillator and (ii) $R-L-C$ oscillator with $R = [3L/C]^{\frac{1}{2}}$ i.e., $CR^2 = 3L$. For case (i), $\partial\omega/\partial R = -\sqrt{3}/CR^2$ as already seen. For case (ii), substituting $\omega = \sqrt{3}/CR$ and $L = CR^2/3$ in Eqn. (12) gives

$$\left(\frac{\partial\omega}{\partial R}\right)_{RLC} = -\frac{\sqrt{3}}{\frac{5}{3}CR^2} = \frac{3}{5}\left(\frac{\partial\omega}{\partial R}\right)_{RC} \qquad \dots \ (15)$$

This shows that for the same value of ω with a particular value of R and C the stability of frequency with respect to variation in R is higher if an inductance $L = CR^2/3$ is included in series with the anode load resistance.

To minimise this type of frequency variation it is obviously best to work in the region where $\partial\omega/\partial R = 0$. Since we have two Eqns. (13a and 13b) and three quantities R_{max} , L and C to be estimated for the desired value of ω_{max} to be maintained, we have a wide choice in the selection of these parameters. From the point of view of reduction in frequency variation due to changes in valve inter-electrode capacitances we should select as large a value of C as is possible.
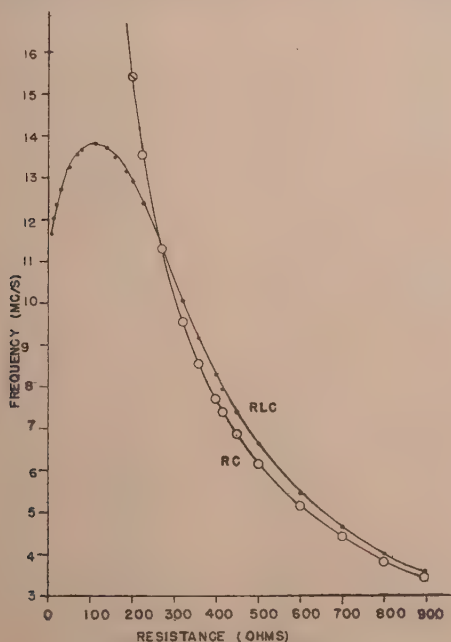

Fig. 3. Variation of Frequency with Resistance.

Thus if the desired frequency is 20 Mc/s. and is to be obtained with $C = 100\mu\mu F$. Eqn. (13a) gives L is of the order of $0.964\mu H$ and Eqn. (13b) gives R_{max} is of the order of 68.94 ohms.

The frequencies calculated according to Eqn. (3) for different values of R and (i) with fixed values of L and C for $R-L-C$ oscillator and (ii) fixed value of C for $R-C$ oscillator are given in Table III and the variations of frequency with R are plotted in Fig. 3. The curves in Fig. 3 further show that for the typical values of L and C chosen in the example although $\partial\omega/\partial R$ is negative in either case for values of $R > [L/2C]^{\frac{1}{2}}$, the magnitude of $\partial\omega/\partial R$ is lower for the RLC case for moderate values of R. As R is further increased $\partial\omega/\partial R$ becomes equal for the two cases and for still larger values of R, $\partial\omega/\partial R$ for the RLC case becomes higher than that for the RC case. Again for very high resistances the slopes tend towards equality.

TABLE III

Variation of frequency with R

$$C = 89.4\mu\mu F; \quad L = 2.2277\mu H$$

R in ohms. (Effective value)	Frequency in Mc/s		$f_{R-L-C} - f_{R-C}$ in Mc/s.
	$R-L-C$	$R-C$	
5	11.672		
13.5	12.02		
22	12.357		
32	12.717		
52	13.237	Oscillation is not possible with valves having g_m not greater than 10mA/V	
70	13.55		
80	13.663		
100	13.802		
111.6	13.812		
138	13.723		
160	13.505		
185	13.413		
200	12.91	15.417	−2.507
228	12.348	13.520	1.172
273.5	11.27	11.27	0
323	10.044	9.562	+0.482
360	9.159	8.565	0.594
400	8.296	7.709	0.587
416	7.930	7.403	0.527
450	7.370	6.852	0.518
500	6.613	6.617	0.446
600	5.427	5.139	0.302
700	4.602	4.404	0.198
800	3.994	3.854	0.140
900	3.527	3.429	0.098
1000	3.155	3.083	0.072

(b) *Variation of ω with L.*

This case applies only to the $R-L-C$ oscillator and proceeding as already discussed we get

$$\frac{\partial \omega}{\partial L} = -\frac{\partial F}{\partial L} \Big/ \frac{\partial F}{\partial \omega} = \frac{\omega(1-2LC\omega^2)}{3\omega^2 L^2 C + (CR^2 - L)} = \frac{\omega(1-2LC\omega^2)}{2\omega^2 L^2 C + \dfrac{\sqrt{3}R}{\omega}} \quad \cdots \quad (16)$$

Here also, as in case of variation of ω with R, the nature of variation of ω is dependent upon whether $2LC\omega^2$ is greater or less than unity. For negligibly small values of L, $\partial\omega/\partial L$ is positive and with increase of L the rate is gradually reduced to zero and thereafter $\partial\omega/\partial L$ becomes negative. The maximum value of ω when $\partial\omega/\partial L = 0$ is given by $\omega_{max} = 1/[2CL_{mdx}]^{\frac{1}{2}}$, where L_{max} is the value of L for which $\partial\omega/\partial L = 0$. Substituting this value of L_{max} in Eqn. (2), we get

$$4\omega^2 C^2 R^2 - 4\sqrt{3}\omega CR - 1 = 0$$

$\therefore \quad \omega_{max} = (\sqrt{3} \pm 2)/2CR$. The negative sign is obviously inadmissible and hence

$$\omega_{max} = 1.866/CR \text{ and } L_{max} = 0.1435 \ CR^2 \quad\quad \cdots \quad (17)$$

After reaching maximum frequency for $L = L_{max}$, $\partial\omega/\partial L$ becomes negative with further increase in L. At a certain value of L greater than L_{max} the fre-
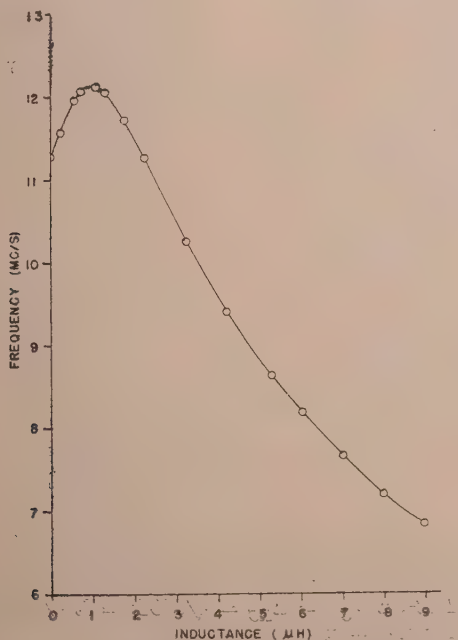

Fig. 4. Variation of frequency with inductance.

quency becomes equal to that of a simple $R-C$ oscillator. This value of L is, as before, given by $CR^2 = 3L$.

The frequencies calculated according to Eqn. (3) for different values of L and with fixed values of R and C are given in Table IV and the results are plotted in Fig. 4.

TABLE IV

Variation of frequency with L

$$C = 89.4\mu\mu F; \ R = 273.5 \ \Omega$$

Inductance in μH	Frequency in Mc/s.	Inductance in μH.	Frequency in Mc/s.
0.1937	11.580	6	8.1713
0.5444	11.979	7	7.665
0.7087	12.085	8	7.2042
0.9693	12.147	9	6.834
1.062	12.136	11.00	6.216
1.30	12.06	13.374	5.6063
1.749	11.725	15.00	5.3179
2.2277	11.27	17.00	4.979
3.2386	10.26	20.00	4.5958
4.1824	9.4155	30.00	3.677
5.2923	8.6261	40.00	3.1776

(c)   *Variation of ω with C.*

1.   In case of $R-C$ oscillator $\omega = \sqrt{3}/CR$ and

$$\frac{\partial \omega}{\partial C} = - \frac{\sqrt{3}}{C^2 R} \qquad \text{... (18)}$$

and is always negative.

2.   In case of the $R-L-C$ oscillator,

$$\frac{\partial \omega}{\partial C} = - \frac{\partial F}{\partial C} \Big/ \frac{\partial F}{\partial \omega} = - \frac{\omega^3 L^2 + \omega R^2}{2\omega^2 L^2 C + \dfrac{\sqrt{3}R}{\omega}} \qquad \text{... (19)}$$

and is always negative. When C is negligibly small, Eqn. (2) reduces to $\omega L + \sqrt{3}R = 0$, i.e. oscillations are not possible with very small values of C. At a value of $C = 3L/R^2$, $\omega_{LC} = \omega_{RC} = \sqrt{3}/CR = R/\sqrt{3}L$, and substituting $\omega = \sqrt{3}/CR$ and $L = CR^2/3$ in Eqn. (19) we get

$$\frac{\partial \omega}{\partial C} = - \frac{4}{5} \frac{\sqrt{3}}{C^2 R} = \frac{4}{5} \left(\frac{\partial \omega}{\partial c} \right)_{RC} \qquad \text{... (20)}$$

This shows that with a particular combination of R and C, the stability of frequency with respect to variation in C is higher if an inductance $L = CR^2/3$ is included in series with the anode load resistance. Equations (15) and (20) show that this inductance improves frequency stability with regard to variations in R as well as C.
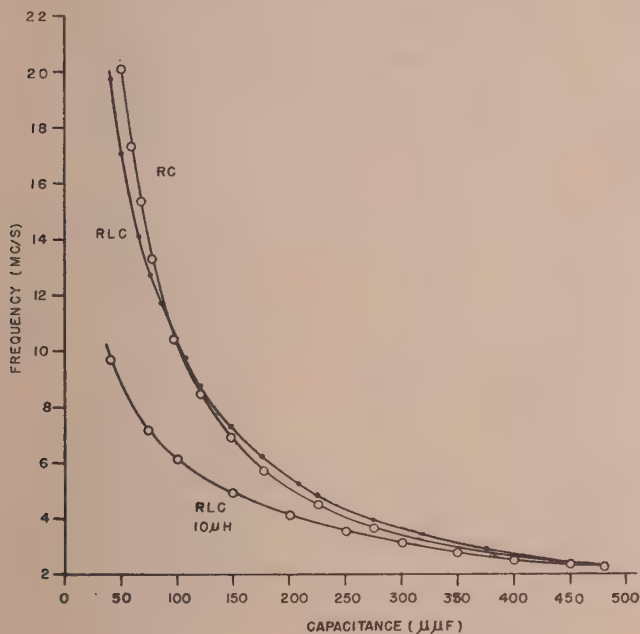

Fig. 5. Variation of frequency with capacitance.

The frequencies calculated according to Equ. (3) for different values of C and (i) with fixed values of L and R for $R-L-C$ oscillator and (ii) with fixed value of R for the $R-C$ oscillator are given in Table V and the results are plotted in Fig. 5. In the same figure the variation of frequency with capacity for same R but $L = 10\mu H$ is also plotted.

EFFECT OF CATHODE CIRCUIT IMPEDANCE ON OSCILLATOR FREQUENCY

The impedance of the cathode circuit of an oscillator valve may be resistive, inductive or capacitive depending on the magnitude of cathode biasing capacitance and operating frequency. The nature of impedance of the cathode circuit at different frequencies with different circuit components has been discussed

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in Part I in connection with $R-C$ oscillator with non-inductive load resistance. It has been pointed out there that the effect of cathode impedance Z_k on the oscil-

TABLE V

Variatiln of frequency with C
$R = 273.50\Omega$; $L = 2.2277\mu H$

Capacitance in $\mu\mu F$	Frequency in Mc/s.		Capacitance in $\mu\mu F$.	Frequency in Mc/s.	
	$R-L-C$	$R-C$		R $L-C$	$R-C$
40	19.72	25.19	120	8.743	8.420
49.5	17.05	20.04	133	7.975	7.578
58	15.355	17.35	147	7.276	6.856
66	14.039	15.27	175	6.184	5.772
76	12.665	13.26	207.4	5.236	4.860
85	11.633	11.857	225	4.821	4.478
89.4	11.27	11.27	275	3.937	3.664
96	10.556	10.499	375	2.845	2.694
106	9.724	9.509	480	2.209	2.120

lator load is to change its effective value. When, for example, the screen is decoupled to ground the anode load Z_0 is changed to (Sturley, 1949)

$$Z_{eff} = \frac{Z_0}{1+g_k Z_k} \qquad \qquad \dots \quad (21)$$

where $g_k = g_m + g_s$, g_s being screen current-grid voltage slope conductance.

The presence of some impedance in the cathode circuit also influences the input admittance of the valve which becomes perceptible only at high frequency. In the present case the effect of change in input admittance on the oscillator frequency which is very small, has been ignored.

Substituting the value of Z_0 from Eqn. (1) we find, on simplification, that the phase angle θ of the effective load is given by

$$\tan \theta = \frac{\omega\{L - \omega^2 L^2 C - CR^2\}\{1 + g_k r\} - g_k XR}{R(1 + g_k r) + \omega X g_k (L - \omega^2 L^2 C - CR^2)} \qquad \dots \quad (22)$$

where r and X are the resistive and reactive parts of the cathode impedance. The frequency of the oscillation is given by $\tan \theta = -\sqrt{3}$

Or $\qquad -\sqrt{3} = \dfrac{\{\omega L - \omega^3 L^2 C - \omega CR^2\}\{1 + g_k r\} - g_k XR}{R(1+g_k r) + \omega X g_k(L-\omega^2 L^2 C - CR^2)} \qquad \dots \quad (23)$

Near about the resonant frequency of the cathode circuit the magnitude of the reactive component X is negligible and then the frequency of oscillation is given by

$$-\sqrt{3} = \frac{\{\omega L - \omega^3 L^2 C - \omega C R^2\}\{1 + g_k r\}}{R(1 + g_k r)}$$

or
$$\omega^3 L^2 C + \omega(CR^2 - L) - \sqrt{3}R = 0$$

which, as expected, is the same as Eqn. (2).

When the generated frequency is greater than the resonant frequency of the cathode circuit the cathode impedance will be inductive and we may put $X = \omega_1 L'$, where

$L' =$ effective inductance of the cathode circuit which is also a function of the generated frequency ω_1.

Equation (23) then reduces to

$$\sqrt{3}\omega_1{}^4 L^2 C L' g_k + \omega_1{}^3 L^2 C(1 + g_k r) + \omega_1(CR^2 - L)(1 + g_k r)$$

$$- \sqrt{3}R(1 + g_k r) + \sqrt{3}\omega_1{}^2 L' g_k(CR^2 - L) + \omega_1 L' g_k R = 0 \qquad \ldots \quad (24)$$

This being a bi-quadratic equation in ω_1 it is not possible to have a simple solution. It can however be shown that when the cathode circuit is inductive the generated frequency decreases. This can be seen by comparing Eqn. (24) which may be put in the form

$$\omega_1{}^3 L^2 C + \omega_1(CR^2 - L) - \sqrt{3}R = -\frac{4\omega_1 L' g_k R}{1 + g_k r + \sqrt{3}\omega_1 L' g_k} \qquad \ldots \ldots \quad (24a)$$

and Eqn. (2) according to which the generated frequency is ω when the cathode circuit is resonant, given by

$$\omega^3 L^2 C + \omega(CR^2 - L) - \sqrt{3}R = 0$$

Since the right hand side of Eqn. (24a) is essentially a negative quantity, $\omega_1 < \omega$ because the anode circuit parameters L, C and R are maintained constant.

The absolute difference between ω and ω_1 naturally depends upon the magnitude of $4\omega_1 L' g_k R/(1 + g_k r + \sqrt{3}\omega_1 L' g_k)$. When either g_k or R or both are small $\omega - \omega_1$ is also small, i.e. the influence of cathode circuit on the oscillator frequency will be small.

Similarly when the cathode circuit is capacitive i.e. the generated frequency is less than the resonant frequency of the cathode circuit, we may put $X = -1/\omega_2 C'$ where $C' =$ effective capacitance of the cathode circuit which is of

course a function of the generated frequency ω_2. Equation 23 then reduces to

$$\omega_2{}^3 L^2 C + \omega_2(CR^2 - L) - \sqrt{3}R = \frac{4g_k R}{\omega_2 C'(1 + g_k r) - \sqrt{3}g_k} \qquad \dots \quad (25)$$

The right hand side of Eqn. (25) may be positive or negative depending mainly on the magnitude of C'. It may be mentioned here that in any practical case C' never reaches such a low value as to make $\omega_2 C'(1 + g_k r)$ less than $\sqrt{3}g_k$. The right hand side of Eqn. (25) will then be positive. Now comparing Eqn. (25) with Eqn. (2) we at once see that $\omega_2 > \omega$, since the anode circuit parameters L, C and R are constant. The absolute difference between ω_2 and ω being dependent on the magnitude of $4g_k R/\{\omega_2 C'(1 + g_k r) - \sqrt{3}g_k\}$, when either g_k or R or both are small, $\omega_2 - \omega$ is also small, i.e. the influence of cathode circuit on oscillator frequency will be small.

CONCLUSION

The effect of an added inductance in series with the load resistance of the three-phase oscillator, and in such case the effect of cathode impedance, on the frequency of oscillation has been discussed in detail in the present paper. An interesting feature which has been observed is that $\partial\omega/\partial R$ and $\partial\omega/\partial L$ may have both positive and negative values. These features can be utilized to eliminate frequency variation of the oscillator due to change in temperature. The effect of cathode impedance on the oscillator frequency is very small unless the cathode circuit resonant frequency is far off from the generated frequency and Q factor of the anode circuit is very low. The conclusions arrived at different stages of this paper have been verified experimentally. The results of such observations and detailed design procedure for better frequency stability of the oscillator are being communicated separately.

APPENDIX I

$$\omega = [P + (P^2 + Q^3)^{1/2}]^{1/3} + [P - (P^2 + Q^3)^{1/2}]^{1/3}$$

where

$$P = \frac{\sqrt{3}R}{2L^2 C} \quad \text{and} \quad Q = \frac{CR^2 - L}{3L^2 C} \quad .$$

When Q is positive and $P^2 << Q^3$,

$$[P + (P^2 + Q^3)^{1/2}]^{1/3}$$

$$= \left[P + Q^{3/2} \left\{ 1 + \frac{1}{2} \frac{P^2}{Q^3} - \frac{1}{8} \frac{P^4}{Q^6} + \quad \dots \quad \right\} \right]^{1/3}$$

$$= Q^{1/2} \left[1 + \frac{P}{Q^{3/2}} + \frac{1}{2} \frac{P^2}{Q^3} - \frac{1}{8} \frac{P^4}{Q^6} + \quad \dots \quad \right]^{1/3} ,$$

neglecting other terms which are insignificant in magnitude.

$$= Q^{\frac{1}{2}} \left[1 + \frac{1}{3} \left(\frac{P}{Q^{3/2}} + \frac{1}{2} \frac{P^2}{Q^3} - \frac{1}{8} \frac{P^4}{Q^6} \right) - \frac{1}{9} \left(\frac{P}{Q^{3/2}} + \frac{1}{2} \frac{P^2}{Q^3} - \frac{1}{8} \frac{P^4}{Q^6} \right)^2 \right.$$

$$\left. + \frac{5}{81} \left(\frac{P}{Q^{3/2}} + \frac{1}{2} \frac{P^2}{Q^3} - \frac{1}{8} \frac{P^4}{Q^6} \right)^3 \cdots \right]$$

is of the order of $Q^{\frac{1}{2}} \left[1 + \frac{1}{3} \frac{P}{Q^{3/2}} + \frac{1}{18} \frac{P^2}{Q^3} - \frac{4}{81} \frac{P^3}{Q^{9/2}} + \frac{5}{216} \frac{P^4}{Q^6} \right]$.

Similarly $\qquad [P - (P^2 + Q^3)^{\frac{1}{2}}]^{1/3}$

is of the order of $-Q^{\frac{1}{2}} \left[1 - \frac{1}{3} \frac{P}{Q^{3/2}} + \frac{1}{18} \frac{P^2}{Q^3} + \frac{4}{81} \frac{P^3}{Q^{9/2}} + \frac{5}{216} \frac{P^4}{Q^6} \right]$

$\therefore \quad \omega = Q^{\frac{1}{2}} \left[\frac{2}{3} \frac{P}{Q^{3/2}} - \frac{8}{81} \frac{P^3}{Q^{9/2}} \right]$ approximately

$$= \frac{2}{3} \frac{P}{Q} \left(1 - \frac{4}{27} \frac{P^2}{Q^3} \right) .$$

Substituting the values of P and Q

$$\omega = \frac{\sqrt{3}R}{CR^2 - L} \left[1 - \frac{3R^2 L^2 C}{(CR^2 - L)^3} \right]$$

APPENDIX II

When R is very small, $CR^2 << L$ and $P^2 << Q^3$

Under these conditions $\cos \phi = \frac{P}{Q^{3/2}} = 0$

and we may put $\phi = \frac{\pi}{2} - \beta$, where β is very small.

Then $\qquad \cos \frac{\phi}{3} = \cos \left(30° - \frac{\beta}{3} \right)$ is of the order of $\frac{\sqrt{3}}{2} + \frac{1}{2} \frac{\beta}{3}$

Now $Q = -\frac{1}{3LC} \left(1 - \frac{CR^2}{L} \right)$ and $\left| Q^{3/2} \right|$ is of the order of $\frac{1}{(3LC)^{3/2}} \left(1 - \frac{3}{2} \frac{CR^2}{L} \right)$

$\therefore \quad \left| \frac{P}{Q^{3/2}} \right| \quad \frac{\sqrt{3}R}{2L^2C} \cdot \frac{(3LC)^{3/2}}{1 - \frac{3}{2} \frac{CR^2}{L}}$ is of the order of $\frac{9R}{2} \sqrt{\frac{C}{L}} \left(1 + \frac{3}{2} \frac{CR^2}{L} \right)$.

Neglecting $\dfrac{3}{2}\dfrac{CR^2}{L}$ which is very small in comparison to 1,

$$\cos\phi = \sin\beta = \frac{9R}{2}\sqrt{\frac{C}{L}} \quad\text{or}\quad \frac{\beta}{3} = \frac{3R}{2}\frac{L}{C}$$

and hence $\quad \cos\dfrac{\phi}{3} = \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\cdot\dfrac{3R}{2}\sqrt{\dfrac{C}{L}} = \dfrac{1}{2}\left\{\sqrt{3} + \dfrac{3R}{2}\sqrt{\dfrac{C}{L}}\right\}$

Substituting this value of $\cos\phi/3$ in Eqn. (6)

$$\omega = \frac{1}{\sqrt{LC}} + \frac{\sqrt{3}R}{2L}$$

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Letters to the Editor

1

DIPOLE MOMENTS OF LONG CHAIN DICARBOXYLIC ACIDS

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The heterodyne beat apparatus used in our measurements has been designed and constructed by a modification of the circuit used by Stranathan (1934) and Terman (1947). The apparatus has been standardised by determining the dipole moments of known compounds and the values obtained are found to be in good agreement with the literature values.

All the substances are crystallised twice from alcohol and their purity checked by their m.p. The solvent p-dioxane is purified by the method adopted by Rieche and Milas (1955). The dipole moment is calculated by the following equation deduced by Palit (1952) from the data given in Table I.

$$p_{2_\mu} = \left[\frac{3(\epsilon_1 - n_1^2)}{d_1(\epsilon_1 + 2)(n_1^2 + 2)} \left(1 - \frac{\beta_0}{d_1}\right) \right] + \left[\frac{3\alpha_0}{d_1(\epsilon_1 + 2)^2} \right] - \left[\frac{6n_1\gamma_0}{d_1(n_1^2 + 2)^2} \right]$$

TABLE I

Solvent : p-Dioxane   Temp : 35°C.

Substance	Formulae	α_0	β_0	ν_0	10^{-18} estu
Adipic acid	$(CH_2)_4.(COOH)_2$	3.4975	0.12434	-0.44899	2.50
Azeleic acid	$(CH_2)_7.(COOH)_2$	3.9027	0.89863	-0.17564	2.71
Sebacic acid	$(CH_2)_8.(COOH)_2$	3.9999	0.09292	-0.01901	2.49
Brasyllic acid	$(CH_2)_{11}.(COOH)_2$	3.0822	0.49372	-0.09169	2.68
Tridecane 1 : 13 dicarboxylic acid.	$(CH_2)_{13}.(COOH)_2$	2.8185	0.01382	-0.19709	2.35
Hexadecane 1 : 16 dicarboxylic acid.	$(CH_2)_{16}.(COOH)_2$	2.6858	0.01691	0.009438	2.75

where α_0, β_0 and γ_0 are the concentration coefficients of ϵ, d and n respectively which were obtained by statistical least square evaluation for these quantities.

The observed dipole moment for the homologous series of dicarboxylic acids are found to be nearly a constant. This clearly shows that the dipole moment of dicarboxylic acids are independent of the number of (CH_2) groups attached in between the carboxylic acid groups. The dipole moment of adipic acid which we have determined is 2.50 in accordance with the homologous series of dicarboxylic acids where as the dipole moment of adipic acid given by Tseng-Sun-Yao (1944) is 4.04.

Thanks are due to The Director and Dr. S. S. Bhattacharya of National Chemical Laboratory, Poona, for supplying materials.

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BOOK REVIEWS

ELECTROMAGNETIC ISOTOPE SEPARATORS AND APPLICATIONS OF
ELECTRO-MAGNETICALLY ENRICHED ISOTOPES—By J. Koch
(Editor), R. H. V. M. Dawton, M. L. Smith and W. Walcher. Pp. 314+ix.
North-Holland Publishing Company. Amsterdam. 1958. Price 26.5 guilders.

This book gives an account of gradual development of the technique for
separation of stable and radioactive isotopes in large quantities and also
reveals the details of some methods which were guarded as close secrets during
the last year. Starting from the earliest electromagnetic isotope separator
constructed in 1934 by Oliphant, Shire and Crowther which provided isotopic
samples of Li^6 and Li^7, the authors proceed to describe other experiments
for the separation of stable isotopes including the separation of U^{235} and U^{238}
by Nier, Booth, Dunning and Grosse in 1940. In Chapter III a historical account
of the development of the technique for the separation of artificial radioactive
isotopes has been given. The assignment of mass number to half life of some
fission isotopes by Hayden has also been discussed in this chapter. Next, the
recent development of isotope separators for preparing stable and radioactive
samples for nuclear research has been discussed in detail. Details of design
of several such separators installed in laboratories in different places such as
Stockholm, Uppsala, Gothenburg, Salcacy, etc., have been given andpho to-
graphs of the separators have been reproduced. The history of the United States
and British projects for installation of plants for production of isotopes has been
given in Chapter V. In the next few chapters photographs of such alpha- and
beta-type production units have been reproduced and details of the construction
of 180° and 90° separators at Harwell have been discussed with the help of
illustrations.

Chapter IX dealing with operational experience in the electromagnetic
separation of isotopes gives very useful information regarding the method of
separation of different types of isotopes, such as gaseous, alkali and alkaline earth
metals, elements with very volatile chlorides, sulphur, halogens, lanthanides, etc.
A chapter has been devoted to application of electromagnetically enriched stable
isotopes.

In part C comprising the last three chapters, Dr. W. Walcher has discussed
the general treatment of some problems related to the design of modern isotope
separators, such as mass analysing field systems, production of high current ion
beams, space charge neutralisation in ion beams, etc. A complete bibliography
has been given at the end of each chapter.

It will appear from the above brief review that the publication of this volume is a great boon to experimental physicists engaged in investigations in nuclear physics and the authors will certainly earn the gratitude of numerous workers in this field in different laboratories all over the world for taking the trouble of compiling and publishing this extremely useful store of information the part of which was not available in any existing literature. It may be safely predicted that every laboratory interested in nuclear physics will try to procure and use this book as a reference volume.

S.C.S.

PROCEEDINGS OF THE SYMPOSIUM ON ELECTRONIC WAVE GUIDES. Edited by Jerome Fox, New York., April, 1958. Pp. 418+xix. Interscience Publishers Inc. 250 Fifth Avenue, New York. N.Y.1. Price $5.00.

This volume constitutes a full record of the Proceedings indicated by the title and includes besides the original papers all the addresses and lectures delivered in the symposium arranged by the Microwave Research Institute of the Polytechnic Institute of Brooklyn and held in April, 1958. The topics of the symposium have been divided under five headings, *viz.*, State of the art, Background and recent developments, Plasmas, Mode and large signal theories, and Special phenomena and devices. There was also a round table discussion on 'Electron tubes and solid state devices—future trends' in the afternoon session on the last day of the symposium.

There are three papers on Traveling-wave tubes, New microwaves devices and Dynamic of plasmas and electron beams respectively under the first topic and four similar papers under the second heading mentioned above. Six full papers have been contributed under the topic plasmas. Similarly, there are six papers under the heading Mode and large signal theories and six papers under Special phenomena and devices. In the paper entitled 'Travelling-wave tubes' Doehler has summarised the work on the application of principle of space charge waves in the manufacture of some travelling wave tubes.

Under the title 'New microwave devices" Kompfner has discussed the development of amomonia 'maser' and 'mavar' amplifiers for microwaves. In a third paper read in this session by Gross who has reviewed the work on dynamics of electron beams and plasmas in which one-particle velocity distribution functions have been described, a new collective method in which the ionised gas is treated in terms of density fluctuations has been discussed.

In the next session Marcuvitz read a paper entitled 'General electronic wave guides' in which he reviewed the analyses of propagation and diffraction

problems and the application of guided wave theory in which analysis in terms of cold modes is involved. He then presented an analysis in terms of orthogonal 'hot modes'. In another paper entitled "Electron beam waves in microwave tubes Haus reviewed the work on wave propagation along electron beams and interpreted the small-signal power theorem and the theorem for longitudinal beams and deduced the equations of the magnetron amplifier.

In a third paper Buneman has reviewed the work on the hydrodynamical theory of electron clouds and its application to axis symmetric or planer-startified systems. Similar work on fully randomized multi stream hot plasmas has then been reviewed. In the fourth paper of this session Allis has discussed the work on the theory of plasma oscillations.

In the next session Bradshaw discussed the results on experimental investigation on the phase shift and absorption experienced by a TEM microwave signal in transmission through a relatively long coaxial region containing space charge. Agdur discussed the theory of propagation of guided microwaves through an electron gas in the presence of a static magnetic field. In a third paper Whitmer discussed the theory of non-linear interaction of an electromagnetic wave with an anisotropic plasma and also the results of some experimental investigations in this line. In this session Rydbeck delivered a lecture on interaction between space charge waves and electromagnetic waves in an inhomogeneous ionized medium which has not been reproduced. In the fourth paper Gould and Trivelpiece discussed a new mode of wave propagation on electron beams. In the next paper Smullin and Chorney discussed the theory of propagation of electromagnetic waves in ion loaded wave guides.

In the first paper of the next session by Bernashevsky, Voronov, Iziumova and Tchernov the results of an experimental investigation of double stream amplifiers in the metre and 10 cm range have been discussed. In the next paper read in the session Hok discussed the theory of electrokinetic and electromagnetic noise waves in electronic waveguides and in another paper Kino discussed the theory of mode properties of passive transmission systems. The fourth paper of this session by Klüver deals with a representation of the boundary conditions at the surface of a slow wave circuit and in the fifth paper Wang and McIsaac discuss a fundamental formulation of interaction equation in electronic wave guides. The sixth paper of this session by Rowe deals with the analysis of non-linear 0-type backward-wave oscillations.

In the next session the first paper read by Tchernov deals with the interaction of electromagnetic waves and electron beams in centrifugal electrostatic focusing systems. Feinstein read a paper on emission of space charge wave energy into electromagnetic radiation and another paper on space and time harmonics in electron beams was read by Muller. The theory of interaction

between an electron system and arc discharge plasma has been dealt with by Boyd, Field and Gould in the fourth paper of this session. In the next paper by Ettenberg and Targ the results of an experimental investigation on plasma and cyclotron oscillations have been discussed and the last paper by Weibel describes some new principles for achieving interactions of electrons with radiation under high magnetic field of the order of 1,000,000 gauss.

It is evident that the papers read in the symposium cover the whole field of up-to-date development of theory and practical application of microwave technique and the Proceedings will be extremely useful to research workers in this line.

S.C.S.

IMPORTANT PUBLICATIONS

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